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# UNIT 1: THE MATHEMATICS CURRICULUM FOR UPPER PRIMARY

# 1.1 What is curriculum?

What are the main goals of mathematics education in schools? Simply stated, there is one main goal—the mathematisation of the child's thought processes. In the words of David Wheeler, it is "more useful to know how to mathematise than to know a lot of mathematics".

Curriculum can be broadly understood as the subjects and materials to be taught by an educational institution; typically, the curriculum includes the whole educational experience of the pupils under the guidance of a teacher. It includes what is to be taught and how it is taught.

The term **mathematics curriculum** refers to all instructionally related mathematical experiences of the pupil under the guidance of a teacher. This curriculum seeks to provide the child with a mathematical education that is developmentally appropriate as well as socially relevant. The mathematics programme in each school should be sufficiently flexible to accommodate children of differing levels of ability and should reflect their needs.

There are different types of curriculum; official, formal, informal, hidden, null, actual, enacted, experienced, unintended, core, experience-oriented, subject-oriented, objective-based, and standard-based, amongst others. However, for the purpose of this course two main types of curriculum shall be considered. These are

- 1) Objective-Based Curriculum; and
- 2) Standards-Based Curriculum.

#### **Objective-based Curriculum (OBC)**

Objectives are directions about what educators want the students to be able to do as a result of instruction. Objectives aid students, teachers, and parents by specifying the direction of the curriculum and goals. Objectives are therefore considered essential to goal setting and planning of curricula. Thus, in a whole, objectives help in ensuring that educational processes are aligned with instructional activities being directed towards the defined outcomes of learning.

This type of curriculum describes the end-points or desired outcomes of the curriculum, a unit, a lesson plan, or a learning activity. Objective-based curriculum (OBC) or objective-based

education (OBE) is teaching and learning targeting outcomes of knowledge, competence and orientation (Brandt, 1992 as cited in Chan & Chan, 2009). They specify and describe curriculum outcomes in more specific terms than goals or aims. The immediate past pre-tertiary curriculum that phased out at the 2018/19 academic year in Ghana was an objective-based curriculum (designed to focus on knowledge) rather than measurable standards (emphasizing competency and driven by reasoning and application).

#### Why the past pre-tertiary education curriculum was objective-based:

- followed the objective-based curriculum design model and was officially defined by subject syllabi.
- 2) emphasized the use of official syllabuses, textbooks and teacher's handbooks, which were the only curriculum materials available to teachers and classroom activities were generally textbooks based, even though there were not enough textbooks for all school learners.
- 3) emphasised an activity-based approach which involved inquiry, creativity, manipulation, collaboration and social interaction but upon implementation, the learning and teaching activities in classrooms tended to favour an expository or didactic teaching approach which was largely teacher-centered.
- 4) Made provision for the use of Ghanaian languages as the medium of instruction for kindergarten and the first three years of primary school through which leaners were to learn the mechanics of reading and writing in their local Ghanaian language, as a necessary prerequisite for introducing learners to a foreign language.
- 5) Made use of School-Based Assessment (SBA) with a focus on Class Assessment Tasks (CATs) as a replacement for continuous assessment in order to make assessment more comprehensive (i.e. to cover more applications and affective qualities).

# **1.2** The objective Model

The objective-based curriculum makes use of the objective model curriculum design that basically contains content that is based on specific objectives. The objectives specify expected learning outcomes in terms of specific measurable behaviour. This behaviour includes;

a. Learning focuses on mastery of content

- b. The teacher is at the centre of the learning process
- c. Overdependence on textbooks, worksheet activities
- d. More concerned with preparation for next grade (or next test)
- e. Less focused on understanding of concepts and mastery of skills by learners

The model is conceptualized in diagram 1

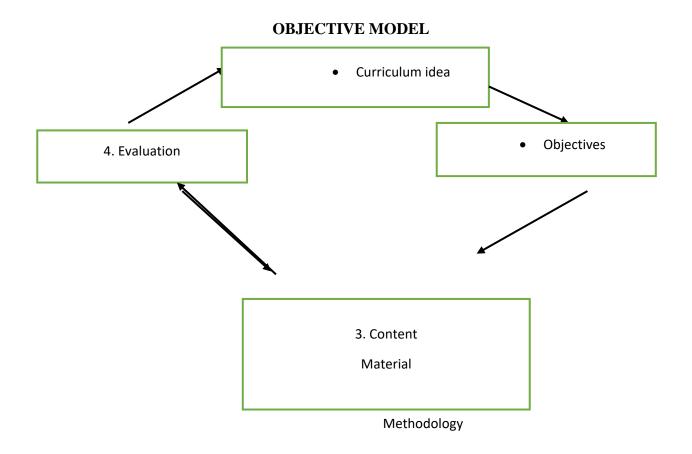


Diagram 1: the objective model

The model when explained clockwise implies that:

- 1) Curriculum organization begins with curriculum idea;
- 2) This is followed by formulation of objectives;
- 3) The next step is selection of content, material and methodology. However, content, material and methodology are derived from the objectives. Also, **content** here means

subject matter and **material** stands for all the resources needed for effective implementation of the curriculum.

4) Then evaluation follows which is equally done at every stage of the curriculum. There are both formative and summative evaluation (IEDE, 2014). It also implies that the use of the objective model of the curriculum led to an over-emphasis on the products of learning; that is, knowing basic facts, principles, skills and procedures at the expense of the processes of learning which involve higher cognitive competences such as applying, thinking critically, creatively and practically; and the personal qualities and social skills necessary to become competent, engaging and contributing citizens.

# **1.3** Advantages of Objective Based Curriculum/Objective-Based Education (OBE)

- 1. Learners know what is exactly expected of them as unit standards make it very clear what is required of them.
- 2. There is greater buy-in and support for OBE from all role-players due to the extensive level of consultation and stakeholder involvement.
- 3. Well-defined assessment criteria makes it clear to both assessors and learners how assessment will take place.
- 4. Assessment is more objective and fair as a result of pre-determined assessment criteria.
- 5. OBE promotes the acquisition of the specific skills and competencies in a country in which there are many skill concepts.
- 6. OBE fosters a better integration between education at school, workplace, and higher education level.
- 7. OBE helps learners to accept responsibility for learning, as they are now at the center of the learning process.
- 8. OBE recognizes prior learning which prevents the duplication and repetition of previous learning situations (Sherill, 2017).

# **1.4 Disadvantages of Objective Based Curriculum**

- 1. The shift to OBE has attracted lots of opposition. Opponents believed that, education should be an open ended and should not be constrained by outcomes and that education should be valued for its own sake, not because it leads to some outcome. They believe that defining education as a set of outcomes where teaching and learning is decided in advance conflicts with the wonderful, unpredictable voyages of exploration that characterize learning through discovery and inquiry. They mistakenly assume that teaching will be inappropriately limited by this model. Moreover they are arguing and emphasizing on what they believe; that the inclusion and emphasis on attitudes and values in stated outcomes is inappropriate (Eldeeb & Shatakumari, 2013; Harden, Crosby & Davis, 1999; Mckernan, 1993).
- Critics of OBE object to the use of standardized tests thinking that it is unfair to use the same level of work or to use the same achievement tests for impoverished or racially disadvantaged students as they do for more advantaged students.
- 3. They also claim that the OBE approach does not permit special, lower standards for students who have been badly served by public education in the past.
- 4. Regarding the outcomes, many opponents dislike them because they think the outcomes' standards maybe too easy, too hard, or wrongly conceived.
- 5. In addition, some critics object to additional resources being spent on the struggling students.
- Finally, some teachers find their grading workload significantly increasing in OBE curriculum (Mckernan, 1993; Eldeeb&Shatakumari, 2013)

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# **Standard-Based Curriculum**

To start with, 'standard' refers explicitly to specific knowledge, learning experiences to gain that knowledge, and assessments to check for mastery of that knowledge, developed by looking at the standards of a district, state, or nation. It is also a statement of what students should know and be able to do and demonstrate at the end of the process at each level. Standards can be used as reference point for planning, teaching and learning programmes and for assessing students' progress.

"A standards-based curriculum refers to a curriculum which has standards to be achieved across the educational system by identifying the knowledge, skills and dispositions that learners should know and be able to demonstrate" (The Pre-Tertiary Education Curriculum Framework, 2018; p.29). It also specifies activities leading to the attainment of specified standards. A Standards Based curriculum is a body of knowledge and set of competencies that form the basis for a quality education. It defines what students should know, understand and be able to do and includes the accompanying teaching content. In the same vein, standard-based curriculum has been defined as "a curriculum that is developed by looking at the standards (district, state or national); identifying the skills, knowledge and dispositions that students should demonstrate to meet these standards; and identifying activities that will allow learners to reach the goals stated in the standards" (Lund & Tannehill, 2014, p.7).

A standards-based curriculum is designed using a Learning Outcomes. The Learning Outcome comprises a series of curriculum outcomes statements describing what knowledge, skills and attitudes learners are expected to demonstrate as a result of their cumulative learning experiences in school. Other terminologies used simultaneously to describe Standard include Learning outcomes, Learning objectives, Learning targets, or Competencies.

In education, the term standard-based refers to systems of instruction, assessment, grading and academic reporting that are based on students demonstrating understanding or mastery of the knowledge or skills they are expected to learn as they progress through their education. Consequently, there are different types of standards that must be ensured in a standard-based curriculum. These include:

1) *Content Standards*: These are statements about what learners should know and be able to do with the contents.

- Performance Standards: These show how the learners have achieved the standard targeted. They show how learners are meeting the standard and their progress in meeting these standards.
- 3) *Proficiency Standards:* These indicate to us how the learners should perform.

The **standards-based curriculum** (SBC) or **the intended curriculum** is therefore the official or adopted curriculum contained in state or district policy. Standards-based curriculum (SBC) describes a body of content knowledge and competencies that students are expected to learn based on their participation within the school experience. Thus the SBC includes broad descriptions of content areas and often specifies performance standards that students are expected to meet and is said to be the basis for quality education. State and district assessments are linked directly to the content and performance standards contained in the SBC. The SBC also outlines graduation requirements, which are taken from state department of education guidelines that specify the subjects and skills that should be taught at each grade level.

Standards-based curriculum helps teachers to link the taught curriculum to the required standards. It is the connection between the content standards and the taught and learned curriculum. The general goal of standards-based learning is to ensure that students are acquiring the knowledge and skills that are deemed to be essential to success in school, higher education, careers, and adult life.

# **1.5 Characteristics of Curriculum Standards**

- a. They are connected to community needs and student needs.
- b. They empower teachers.
- c. They are based on principled procedures.
- d. They are flexible and able to change.
- e. They include timelines for students' learning, development and growth.
- f. They describe a whole curriculum.
- g. Hidden curriculum is considered.
- h. They identify big ideas, concepts and outcomes.

- i. They include assessment Formative, summative, diagnostic and generative.
- j. They include ways to satisfy accountability.

# **1.6** Characteristics of a standards-based classroom

- a. Classroom climate is characterised by respectful behaviours, routines and discourse;
- b. Classroom practices and instructions honour the diversity of interests, needs and strengths of all learners;
- c. The teacher ensures that all components of the lesson (e.g., learning activities, assessment, homework etc.) contribute to the lesson objectives and to the student mastery of the standard(s);
- d. Learning time is maximised for all learners;
- e. Instruction activates learners' prior knowledge and experience, and supplies background knowledge; and
- f. Learners respond to opportunities provided by the teacher to make connections between the lesson and personal experience (Adopted from M'barek El-farhaoui).

# **Distinction between Objective-Based and Standards-Based Curriculum**

- 1) OBC focuses on what learners should know and little about what they should be able to do whereas in the SBC, emphasis is on what the learners should be able to do.
- The content matter of the OBC is pre-determined and specific approaches to teaching are suggested. In the case of SBC, the content is flexible and encourages teachers to engage in innovative teaching to meet the unique needs of learners;
- 3) There is rigid prescriptions of textbooks that are aligned to the various subject syllabi in the OBC. On the other hand, the SBC does not necessary conform to the strict adherence to the behavioural view of learning.

- In OBC, learners are consumers of knowledge whereas in the SBC, learners are creators of knowledge.
- 5) In OBC, learning focuses on mastery of content whereas in the SBC, learners respond to opportunities provided by the teacher to make connections between the lesson and personal experience
- 6) In the OBC, the teacher is at the centre of the learning process whereas in the SBC, the learner are at the centre of the learning process leading to lifelong learning.

#### <u>Activity</u>

- 1. Identify any six (6) types of curriculum design.
- 2. Explain the following curriculum typologies:
  - a) Objective-based curriculum
  - b) Standard-based curriculum
- 3. Analyze any four (4) distinctions/differences between objective-based and standard-based curriculum.

Write any three (3) weaknesses and strengths of each of the aforementioned curriculum designs.

# Levels of the school mathematics curriculum

Sometimes, the curriculum is described from the point of view of what learners actually realize from what is provided. On the basis of what is planned, the International Commission on Mathematical Instruction (ICMI) contended that the content of the school mathematics curriculum can be viewed at three levels (Howson and Wilson, 1986) these are

- (i) The intended curriculum what is prescribed in syllabuses (national/examination) and official text book
- (ii) The implemented curriculum-what teachers teach
- (iii) The attained curriculum- what students learn

# **CURRICULUM MATERIALS**

Curriculum materials include the three basic documents or reference materials available for teachers for effective lesson preparation, and presentation. They are:

- (i) Mathematics Curriculum for Primary School (B4-B6)
- (ii) Teachers Resource Pack
- (iii) Learners Resource Pack

The importance given to mathematics in the school curriculum cannot be over emphasized. The following are some of the reasons why we teach or learn mathematics.

- It helps us to be able to count and make simple calculations with numbers
- It helps us to know about money and be able to make simple calculations involving money.
- It is a necessary basis for further studies in many fields.
- It helps us to recognize shapes and know some of their properties
- It helps us to design and play games like oware, ludo, domino, snake and ladder etc.

Maths is used in science and technology, industry, government, economics etc.

#### FRONT MATTERS OF THE NEW MATHEMATICS CURRICULUM

# **RATIONALE FOR PRIMARY MATHEMATICS**

Mathematics forms an integral part of our everyday lives. is a universal truth that development is hinged on Mathematics is the backbone of social, economic, political and physical development of a country. It is a never-ending creative process which serves to promote discovery and understanding. It consists of a body of knowledge which attempts to explain and interpret phenomena and experiences. Mathematics has changed our lives and is vital to Ghana's future development.

To provide quality Mathematics education, teachers must facilitate learning in the Mathematics classroom. This will provide the foundations for discovering and understanding the world around us and lay the grounds for Mathematics and Mathematics related studies at higher levels of education. Learners should be encouraged to understand how Mathematics can be used to explain what is occurring, predict how things will behave and analyse causes and origins of things in our environment. The Mathematics curriculum has considered the desired outcomes of education for learners at the basic level. Mathematics is also concerned with the development of attitudes. It is important for all citizens to be mathematically and technologically literate for sustainable development. Mathematics therefore ought to be taught using hands-on and minds-on approaches which learners will find as fun and adopt as a culture.

#### PHILOSOPHY

#### **Teaching Philosophy**

Ghana believes that an effective mathematics education needed for sustainable development should be inquiry-based. Thus mathematics education must provide learners with opportunities to expand, change, enhance and modify the ways in which they view the world. It should be pivoted on learner-centred mathematics teaching and learning approaches that engage learners physically and cognitively in the knowledge-acquiring process in a rich and rigorous inquiry-driven environment.

## **Learning Philosophy**

Mathematics learning is an active contextualized process of constructing knowledge based on learners' experiences rather than acquiring it. Learners are information constructors who operate as researchers. Teachers serve as facilitators by providing the enabling environment that promotes the construction of learners' own knowledge, based on their previous experiences. This makes learning more relevant to the learner and leads to the development of critical thinkers and problem solvers.

#### **GENERAL AIMS**

The curriculum is aimed at developing individuals to become mathematically literate, good problem solvers, have the ability to think creatively and have both the confidence and competence to participate fully in Ghanaian society as responsible local and global citizens.

#### SUBJECT AIMS

The mathematics curriculum is designed to help learners to:

1. recognise that mathematics permeates the world around us;

2. appreciate the usefulness, power and beauty of Mathematics;

3. enjoy Mathematics and develop patience and persistence when solving problems;

4. understand and be able to use the language, symbols and notation of Mathematics; and

5. develop mathematical curiosity and use inductive and deductive reasoning when solving problems;

6. become confident in using mathematics to analyse and solve problems both in school and in real-life situations; develop the knowledge, skills and attitudes necessary to pursue further studies in Mathematics; and

8. develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others.

#### **INSTRUCTIONAL EXPECTATIONS**

1. Guide and facilitate learning by generating discourse among learners and challenging them to accept and share responsibility for their own learning, based on their unique individual differences.

2. Select Mathematics content, adapt and plan lessons to meet the interests, knowledge, understanding, abilities and experiences of learners.

3. Work together as colleagues within and across disciplines and grade levels to develop communities of Mathematics learners who exhibit the skills of mathematical inquiry and the attitudes and social values conducive to Mathematics learning.

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4. Use multiple methods and systematically gather data about learner understanding and ability to guide Mathematics teaching and learning, with arrangements to provide feedback to both learners and parents.

5. Design and manage learning environments that provide learners with the time, space and resources needed for learning mathematics

#### **CORE COMPETENCIES**

The core competencies for studies describe a body of skills that teachers in Mathematics at all levels should seek to develop in their learners. They are ways in which teachers and learners in Mathematics engage with the subject matter as they learn the subject. The competencies presented here describe a connected body of core skills that are acquired throughout the processes of teaching and learning.

#### **CRITICAL THINKING AND PROBLEM SOLVING (CP)**

This skill develops learners' cognitive and reasoning abilities to enable them analyse and solve problems. Critical thinking and problem-solving skill enables learners to draw on their own experiences to analyse situations and choose the most appropriate out of a number of possible solutions. It requires that learners embrace the problem at hand, persevere and take responsibility for their own learning.

#### **CREATIVITY AND INNOVATION (CI)**

Creativity and Innovation promotes entrepreneurial skills in learners through their ability to think of new ways of solving problems and developing technologies for addressingthe problem at hand. It requires ingenuity of ideas, arts, technology and enterprise. Learners having this skill are also able to think independently and creatively.

#### COMMUNICATION AND COLLABORATION (CC)

This competence promotes in learners the skills to make use of languages, symbols and texts to exchange information about themselves and their life experiences. Learner's actively participate in sharing their ideas. They engage in dialogue with others by listening to and learning from them. They also respect and value the views of others.

#### CULTURAL IDENTITY AND GLOBAL CITIZENSHIP (CG)

This competence involves developing learners to put country and service foremost through an understanding of what it means to be active citizens. This is done by inculcating in learners a strong sense of social and economic awareness. Learners make use of the knowledge, skills, COMPETENCIES and attitudes acquired to contribute effectively towards the socioeconomic development of the country and on the global stage. Learners build skills to critically identify and analyse cultural and global trends that enable them to contribute to the global community.

#### PERSONAL DEVELOPMENT AND LEADERSHIP (PL)

This competence involves improving self-awareness and building self-esteem. It also entails identifying and developing talents, fulfilling dreams and aspirations. Learners are able to learn from mistakes and failures of the past. They acquire skills to develop other people to meet their

needs. It involves recognizing the importance of values such as honesty and empathy and seeking the well-being of others. Personal development and leadership enables learners to distinguish between right and wrong. The skill helps them to foster perseverance, resilience and selfconfidence. PL helps them acquire the skill of leadership, self-regulation and responsibility necessary for lifelong learning.

#### **DIGITAL LITERACY (DL)**

Digital Literacy develop learners to discover, acquire, and communicate through ICT to support their learning. It also makes them use digital media responsibly.

#### LEARNING DOMAINS (EXPECTED LEARNING BEHAVIOURS)

A central aspect of this curriculum is the concept of three integral learning domains that should be the basis for instruction and assessment. These are

- Knowledge, Understanding and Application
- Process Skills
- Attitudes and Values

#### KNOWLEDGE, UNDERSTANDING AND APPLICATION

Under this domain, learners may acquire some knowledge through some learning experiences. They may also show understanding of concepts by comparing, summarizing, rewriting etc. in their own words and constructing meaning from instruction. The learner may also apply the knowledge acquired in some new contexts. At a higher level of learning behaviour, the learner may be required to analyse an issue or a problem. At a much higher level, the learner may be required to synthesize knowledge by integrating a number of ideas to formulate a plan, solve a problem, compose a story, or a piece of music. Further, the learners may be required to evaluate, estimate and interpret a concept. At the last level, which is the highest, learners, may be required to create, invent, compose, design and construct. These learning behaviours "knowing", "understanding", "applying", "analysing", "synthesising", "evaluating" and "creating" fall under the domain "Knowledge, Understanding and Application".

In this curriculum, learning indicators are stated with action verbs to show what the learner should know and be able to do. For example, the learner will be able to describe something. Being able to "describe" something after teaching and learning has been completed means that the learner has acquired "knowledge". Being able to explain, summarise, and give examples etc. means that the learner has understood the concept taught similarly, being able to develop, defend, etc. means that the learner can "apply" the knowledge acquired in some new context. You will note that each of the indicators in the curriculum contains an "action verb" that describes the behaviour the learner will be able to demonstrate after teaching and learning has taken place. "Knowledge,

#### Understanding

and Application" is a domain that should be the prime focus of teaching and learning in schools. Teaching in most cases has tended to stress knowledge acquisition to the detriment of other higher level behaviours such as applying knowledge.

Each action verb in any indicator outlines the underlying expected outcome. Each indicator must be read carefully to know the learning domain towards which you have to teach. The focus is to move teaching and learning from the didactic acquisition of "knowledge" where there is fact memorization, heavy reliance on formulae, remembering facts without critiquing them or relating them to real world – *surface learning* – to a new position called – *deep learning*. Learners are expected to deepen their learning by knowledge application to develop critical thinking skills, explain reasoning, and to generate creative ideas to

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solve real life problems in their school lives and later in their adult lives. This is the position

where learning becomes beneficial to the learner.

The explanation and the key words involved in the "Knowledge, Understanding and
Application" domain are as follows:

Knowing:	The ability to remember, recall, identify, define, describe, list, name, match, state principles, facts, concepts. Knowledge is the ability to remember or
	recall material already learned and this constitutes the lowest level of learning.
	The ability to explain, summarise, translate, rewrite, paraphrase, give
Understanding:	examples, generalise, estimate or predict consequences based upon a trend.
Chuci standing.	Understanding is generally the ability to grasp the meaning of some material
	that may be verbal, pictorial, or symbolic
	This dimension is also referred to as "Use of Knowledge". Ability to use
Applying:	knowledge or apply knowledge, apply rules, methods, principles, theories, etc.
Apprying.	to situations that are new and unfamiliar. It also involves the ability to produce,
	solve, plan, demonstrate, discover etc.
	The ability to break down material/information into its component parts; to
A a louging	differentiate, compare, distinguish, outline, separate, identify significant points
Analysis:	etc., ability to recognise unstated assumptions and logical fallacies; ability to
	recognise inferences from facts etc.
	The ability to put parts together to form a new whole. It involves the ability to
Synthesizing:	combine, compile, compose, devise, plan, revise, organise, create,
	generate new ideas and solutions etc
	The ability to appraise, compare features of different things and make
T-volvo4in ov	comments or judgment, compare, contrast, criticise, justify, support, discuss,
Evaluating:	conclude, make recommendations etc. Evaluation refers to the ability to judge
	the worth or value of some material based on some criteria.
	The ability to use information or materials to plan, compose, produce,
	manufacture or construct other products. From the foregoing, creation is the
	highest form of thinking and learning skill and is therefore the most important
	behaviour. This unfortunately is the area where most learners perform poorly.
Creating:	In order to get learners to develop critical thinking and behavioural skills
0	beginning right from the lower primary level, it is advised that you do
	your best to help your learners to develop analytic and application skills as we
	have said already

# SKILLS AND PROCESSES

The mathematical method is the means by which a mathematician solves problems or seeks to

gain information about events. Learners should be exposed to situations that

challenge them to raise questions and attempt to solve problems. The more often they are faced

with these challenges, the more likely they are to develop a positive attitude

	This is the skill of using our senses to gather information about objects or events.
<b>Observing</b> This also includes the use of instruments to extend the range of our	
	senses.

Classifying: This is the skill of grouping objects or events based on common characteristics

**Comparing:** This is the skill of identifying the similarities and differences between two or more objects, concepts or processes.

**Communicating:** This is the skill of transmitting, receiving and presenting information in concise, clear and accurate forms verbal, written, pictorial or tabular.

**Reporting:** - graphical

**Predicting:** This is the skill of assessing the likelihood of an outcome based on prior knowledge of how things usually turn out.

**Analysing:** This is the skill of identifying the parts of objects, information or processes, and the patterns and relationships between these parts.

Generating /	This is the skill of exploring all the options, possibilities and alternatives
possibilities	beyond the obvious or preferred one.

Evaluating	This is the skill of assessing the reasonableness, accuracy and quality of information, processes or ideas. This is also the skill of assessing the quality
Dimuning	and feasibility of objects.

Designing This is the skill of Visualizing and drawing new objects or gargets from imagination.

**Measuring:** This is the skill of using measuring instruments and equipment for measuring, reading and making observations.

	This is the skill of evaluating data in terms of its worth: good, bad, reliable,
Interpreting	unreliable; making inferences and predictions from written or graphical data;
interpreting	extrapolating and deriving conclusions. Interpretation is also referred to as
	"Information Handling".

**Recording** This is the skill of drawing or making graphical representation boldly and clearly, well labelled and pertinent to the issue at hand.

**Generalising:** This is the skill of being able to use the conclusions arrived at in an experiment to what could happen in similar situations.

Designing ofThis is the skill of developing hypotheses; planning and designing of<br/>experiments; persistence in the execution of experimental activities;<br/>modification of experimental activities where necessary in order to reach<br/>conclusions. Learners therefore need to acquire positive attitudes,<br/>values and psychosocial skills that will enable them participate actively in<br/>lessons and take a stand on issues affecting them and others.

#### ATTITUDES

To be effective, competent and reflective citizens, who will be willing and capable of solving personal and societal problems, learners should be exposed to situations that challenge them to raise questions and attempt to solve problems. Learners therefore need to acquire positive attitudes, values and psychosocial skills that will enable them participate in debates and take a stand on issues affecting them and others. The mathematics curriculum thus focuses on the development of attitudes and values.

#### The mathematics curriculum aims at helping learners to acquire the following:

- i. Commitment: determination to contribute to national development.
- ii. Tolerance: willingness to respect the views of others.
- iii. Patriotism: readiness to defend the nation.
- iv. Flexibility in ideas: willingness to change opinion in the face of more plausible evidence.
- v. Respect for evidence: willingness to collect and use data on one's investigation, and also have respect for data collected by others.
- vi. Reflection: the habit of critically reviewing ways in which an investigation or observation has been carried out to see possible faults and other ways in which the investigation or observation can be improved upon.
- vii. Comportment conforming to acceptable societal norms.
- viii. Co-operation the ability to work effectively with others.
- ix. Responsibility: the ability to act independently and make decisions; morally accountable for one's action; capable of rational conduct.
- x. Environmental Awareness: being conscious of one's physical and socio-economic surroundings.
- Respect for the Rule of Law: obeying the rules and regulations of the land. The teacher should ensure that learners cultivate the above attitudes and skills as basis for living in the nation as effective citizens.

#### VALUES

At the heart of this curriculum is the belief in nurturing honest, creative and responsible citizens.

As such, every part of this curriculum, including the related pedagogy should be consistent with the following set of values. *Respect:* This includes respect for the nation of Ghana, its institutions and laws and the culture and respect among its citizens and friends of Ghana.

*Diversity:* Ghana is a multicultural society in which every citizen enjoys fundamental rights and responsibilities. Learners must be taught to respect the views of all persons and to see national diversity as a powerful force for nation development. The curriculum promotes social cohesion.

*Equity:* The socio-economic development across the country is uneven. Consequently, it is necessary to ensure an equitable distribution of resources based on the unique needs of learners and schools. Ghana's learners are from diverse backgrounds, which require the provision of equal opportunities to all, and that all strive to care for each other both personally and professionally.

*Commitment to achieving excellence:* Learners must be taught to appreciate the opportunities provided through the curriculum and persist in doing their best in whatever field of endeavour as global citizens. The curriculum encourages innovativeness through creative and critical thinking and the use of contemporary technology.

*Teamwork/Collaboration:* Learners are encouraged to be become committed to team-oriented working and learning environments. This also means that learners should have an attitude of tolerance to be able to live peacefully with all persons.

*Truth and Integrity:* The curriculum aims to develop learners into individuals who will consistently tell the truth irrespective of the consequences. In addition, be morally upright with the attitude of doing the right thing even when no one is watching. Also, be true to themselves and be willing to live the values of honesty and compassion. Equally important, the ethos or culture of the work place, including integrity and perseverance, must underpin the learning processes to allow learners to apply skills and COMPETENCIES in the world of work. The action verbs provided under the various profile dimensions should help you to structure your teaching to achieve desired learning outcomes. Select from the action verbs provided for your teaching, for evaluation exercises and for test construction. Check the weights

of the profile dimensions to ensure that you have given the required emphasis to each of the dimensions in your teaching and assessment.

#### ASSESSMENT

Assessment is a process of collecting and evaluating information about learners and using the information to make decisions to improve their learning.

In this curriculum, it is suggested that assessment is used to promote learning. Its purpose is to identify the strengths and weaknesses of learners to enable teachers ascertain their learner's response to instruction.

Assessment is both formative and summative. Formative assessment is viewed in terms of Assessment *as* learning and Assessment *for* learning. In this curriculum, it is suggested that assessment is used to promote learning. Its purpose is to identify the strengths and weaknesses of learners to enable teachers ascertain their learner's response to instruction.

Assessment is both formative and summative. Formative assessment is viewed in terms of Assessment *as* learning and Assessment *for* learning.

Assessment *as* learning: Assessment as learning relates to engaging learners to reflect on the expectations of their learning. Information that learners provide the teacher forms the basis for refining teaching-learning strategies. Learners are assisted to play their roles and to take responsibility of their own learning to improve performance. Learners set their own goals and monitor their progress.

Assessment *for* learning: It is an approach used to monitor learner's progress and achievement. This occurs throughout the learning process. The teacher employs assessment for learning to seek and interpret evidence which serves as timely feedback to refine their teaching strategies and improve learners' performance. Learners become actively involved in the learning process and gain confidence in what they are expected to learn.

Assessment *of* learning: This is summative assessment. It describes the level learners have attained in the learning, what they know and can do over a period of time. The emphasis

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is to evaluate the learner's cumulative progress and achievement. It must be emphasised that all forms of assessment should be based on the domains of learning. In developing assessment procedures, try to select indicators in such a way that you will be able to assess a representative sample from a given strand. Each indicator in the curriculum is considered a criterion to be achieved by the learners. When you develop assessment items or questions that are based on a representative sample of the indicators taught, the assessment is referred to as a "Criterion-Referenced Assessment". In many cases, a teacher cannot assess all the indicators taught in a term or year. The assessment procedure you use i.e. class assessments, homework, projects etc. must be developed in such a way that the various procedures complement one another to provide a representative sample of indicators taught over a period.

#### SUGGESTED TIME ALLOCATION

A total of ten periods a week, each period consisting of thirty minutes, is allocated to the teaching of mathematics at the Lower Primary level. It is recommended that the teaching periods be divided as follows:x 2 periods per day (two 30-minute periods)

#### **PEDAGOGICAL APPROACHES**

These include the approaches, methods, strategies, appropriate relevant teaching and learning resources for ensuring that every learner benefits from teaching and learning process. The curriculum emphasises the:

- 1. creation of learning-centred classrooms through the use of creative approaches to ensure learner empowerment and independent learning.
- 2. Positioning of inclusion and equity at the centre of quality teaching and learning.
- 3. Use of differentiation and scaffolding as teaching and learning strategies for ensuring that no learner is left behind.
- 4. Use of Information Communications Technology (ICT) as a pedagogical tool.
- Identification of subject specific instructional expectations needed for making learning in the subject relevant to learners.
- 6. Integration of assessment as learning, for learning and of learning into the teaching and learning processes and as an accountability strategy.

7. Questioning techniques that promote deep learning

#### LEARNING-CENTRED PEDAGOGY

The learner is at the centre of learning. At the heart of the national curriculum for change and sustainable development is the learning progression and improvement of learning outcomes for Ghana's young people with a focus on the 4Rs – Reading, wRiting, aRithmetic and cReativity. It is expected that at each curriculum phase, learners would be offered the essential learning experiences to progress seamlessly to the next phase. Where there are indications that a learner is not sufficiently ready for the next phase a compensatory provision through differentiation should be provided to ensure that such a learner is ready to progress with his/her cohort. At the primary school, the progression phases are KG1 to KG2 and B1 to B6.

The Curriculum encourages the creation of a learning centred classroom with the opportunity for learners to engage in meaningful "hands-on" activities that bring home to the learner what they are learning in school and what they know from outside of school. The learning centred classroom is a place for the learners to discuss ideas through the

inspiration of the teacher. The learners then become actively engaged in looking for answers, working in groups to solve problems. They also research for information, analyse and evaluate information. The aim of the learning-centred classroom is to enable learners take ownership of their learning. It provides the opportunity for deep and profound learning to take place.

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#### The teacher as a facilitator needs to create a learning environment that:

- 1. Makes learners feel safe and accepted
- 2. Helps learners to interact with varied sources of information in a variety of ways
- 3. Helps learners to identify a problem suitable for investigation through project work

4. Connects the problem with the context of the learners' world so that it presents realistic opportunities for learning

5. Organises the subject matter around the problem, not the subject

6. Gives learners responsibility for defining their learning experience and planning to solve the problem

7. Encourages learners to collaborate in learning

8. Expects all learners to demonstrate the results of their learning through a product or performance It is more productive for learners to find answers to their own questions rather than teachers providing the answers and their opinions in a learning-centred classroom.

#### INCLUSION

Inclusion is ensuring access and learning for all learners especially those disadvantaged. All learners are entitled to a broad and balanced curriculum in every school in Ghana. The daily learning activities to which learners are exposed should ensure that the learners' right to equal access and accessibility to quality education is met. The Curriculum suggests a variety of approaches that addresses learners' diversity and their special needs in the learning process. When these approaches are effectively used in lessons, they will contribute to the full development of the learning potential of every learner. Learners have individual needs and learning experiences and different levels of motivation for learning. Planning, delivery and reflection on daily learning experiences should take these differences into consideration. The curriculum therefore promotes: 1. learning that is linked to the learner's background and to their prior experiences, interests, potential and capacities.

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learning that is meaningful because it aligns with learners' ability (e.g. learning that is oriented towards developing general capabilities and solving the practical problems of everyday life); and
 the active involvement of the learners in the selection and organization of learning experiences, making them aware of their importance and also enabling them to assess their own learning outcomes.

# DIFFERENTIATION AND SCAFFOLDING

**Differentiation** is a process by which differences (learning styles, interest and readiness to learn) between learners are accommodated so that all learners in a group have best possible chance of learning. Differentiation could be by content, tasks, questions, outcome, groupings and support. Differentiation as a way of ensuring each learner benefits adequately from the delivery of the curriculum can be achieved in the classroom through i) Task ii) Support from the Guidance and Counselling Unit and iii) Learning outcomes.

Differentiation by task involves teachers setting different tasks for learners of different abilities.E.g. in sketching the plan and shape of their classroom some learners could be made to sketch with free hand while others would be made to trace the outline of the plan.

**Differentiation by support** involves the teacher giving needed support and referring weak learners to the Guidance and Counselling Unit for academic support.

**Differentiation by outcome** involves the teacher allowing learners to respond at different levels. Weaker learners are allowed more time for complicated tasks.

**Scaffolding** in education refers to the use of variety of instructional techniques aimed at moving learners progressively towards stronger understanding and ultimately greater independence in the learning process.

It involves breaking up the learning task, experience or concepts into smaller parts and then providing learners with the support they need to learn each part. The process may require a teacher assigning an excerpt of a longer text to learners to read and engaging them to discuss the excerpt to improve comprehension. The teacher goes ahead to guide them through the key words/vocabulary to ensure learners have developed a thorough understanding of the text before engaging them to read the full text.

#### Common scaffolding strategies available to the teacher are:

1. Give learners a simplified version of a lesson, assignment, or reading, and then gradually increases the complexity, difficulty, or sophistication over time

2. Describe or illustrate a concept, problem, or process in multiple ways to ensure understanding

3. Give learners an exemplar or model of an assignment they will be asked to complete

4. Give learners a vocabulary lesson before they read a difficult text

5. Describe the purpose of a learning activity clearly and the learning goals they are expected to achieve

6. Describe explicitly how the new lesson builds on the knowledge and skills learners were taught in a previous lesson

#### INFORMATION AND COMMUNICATION TECHNOLOGY

Information and Communication Technology (ICT) has been integrated into the mathematics curriculum as part of the core of education, alongside reading, writing and numeracy. Thus, the curriculum is designed to use ICT as a teaching and learning tool to enhance deep and independent learning. For instance, the teacher in certain instances is directed to use multimedia to support the teaching and learning process.

ICT has the potential to innovate, accelerate, enrich, and deepen skills. It also motivates and engages learners to relate school experiences to work practices. It provides opportunities for learners to fit into the world of work.

#### Some of the expected outcomes that this curriculum aims to achieve are:

- 1. Improved teaching and learning processes
- 2. Improved consistency and quality of teaching and learning
- 3. Increased opportunities for more learner-centered pedagogical approaches
- 4. Improved inclusive education practices.
- 5. Improved collaboration, creativity, higher order thinking skills
- 6. Enhanced flexibility and differentiated approach of delivery

The use of ICT as a teaching and learning tool is to provide learners an access to large quantities of information online and offline. It also provides the framework for analysing data to investigate patterns and relationships in the geographical context. Once learners have made their findings, ICT can help them organize, edit and print the information in many different ways.

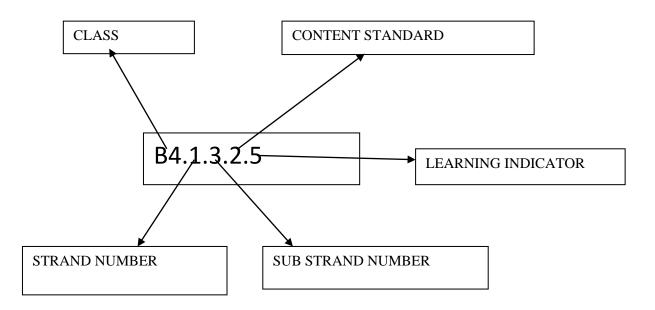
Learners need to be exposed to various ICT tools around them including calculators, radios, cameras, phones, television sets and computers and related software like Microsoft Office packages - Word, PowerPoint and Excel as teaching and learning tools. The exposure that learners are given at the primary school level to use ICT in exploiting learning will build their confidence and will increase their level of motivation to apply ICT use in later years, both within and outside of education. ICT use for teaching and learning is expected to enhance the quality and competence level of learners.

# ORGANISATION AND STRUCTURE OF THE CURRICULUM

The curriculum is organized under key headings and annotations.

## ANNOTATION

A unique annotation is used to label the class, strands, sub-strands, content standards and learning indicators in the curriculum for the purpose of easy referencing. The annotation is defined in figure 1:



## Figure 1: Curriculum Reference Numbers

Strands are the broad areas/sections of the mathematics content to be studied.

Sub-strands are the topics within each strand under which the content is organised.

**Content standard** refers to the pre-determined level of knowledge, skill and/or attitude that a learner attains by a set stage of education.

**Indicator** is a clear outcome or milestone that learners have to exhibit in each year to meet the content standard expectation. The indicators represent the minimum expected standard in a year.

**Exemplar** – support and guidance which clearly explains the expected outcomes of an indicator and suggests what teaching and learning activities could take to support the facilitators/teachers in the delivery of the curriculum.

#### **ORGANIZATION OF THE STANDARDS (B4 – B6)**

The content standards in this document are organized by grade level. Within each grade level, the contents are grouped first by strands. Each strand is further subdivided into sub strands of related indicators.

**A** Indicators are learning outcomes that define what learners should know and be able to do.

• Content Standards are groups of related indicators. Note that indicators from different standards may sometimes be closely related, because mathematics is a connected subject.

\* Sub-strands are larger groups of related indicators (or mathematics topics to be studied).

Indicators from different sub-strands may sometimes be closely related.

**Strands** are the main branches of the mathematics content to be studied.

The Standards are organized at the KG1 – B6 phase under four strands:

1. Number

- 2. Algebra
- 3. Geometry and Measurement
- 4. Data

The table below shows the scope and sequence of the strands addressed at the B4 - B6 phase. The remaining part of the document presents the details of the standards and indicators for each grade level,

STRANDS	SUB-STRANDS		
B4	B5	B6	
Number (Counting, Representation, Cardinality and Operations	Whole Numbers Counting and Representation	Whole Numbers Counting and Representation	Whole Numbers Counting and Representation
Whole Numbers Operations	Whole Numbers Operations	Whole Numbers Operations	
Fractions, Representation and Relationship	Fractions Representation and Relationship	Fractions Representation and Relationship	
Algebra	Patterns and Relationships	Patterns and Relationships	Patterns and Relationships
Functions & Unknowns	Functions & Unknowns	Functions & Unknowns	
Expressions	Expressions	Expressions	
Equations and Inequalities	Equations and Inequalities	Equations and Inequalities	
Geometry and Measurement	Lines and Shapes	Lines and Shapes	Lines and Shapes
Measurements	Measurements	Measurements	
Geometrical Reasoning	Geometrical Reasoning	Geometrical Reasoning	
Data	Data (Collection, organization, interpreting, analysis	Data	Data

# STRUCTURE OF THE CURRICULUM

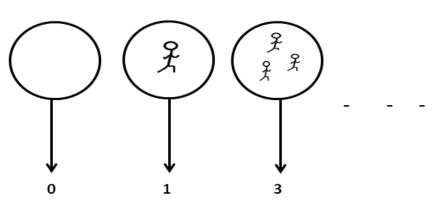
# 2 UNIT 2: COUNTING AND NUMBER RELATIONSHIPS

It is assigning number names to a given objects in the right order.

NB: Counting of numbers is done in 2 main stages

- 1. Rote Counting
- 2. Enumeration counting.

Rote Counting is a way children learn how to count numbers orally or recite – done through the use of rhymes and songs.



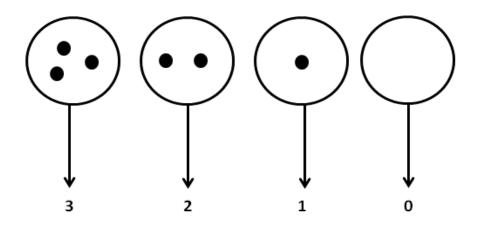
Enumeration is the process of writing objects into definite groups.

After that give an exercise to test that:

#### The Concept of Zero

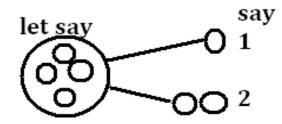
**NB.** Children find it difficult to understand the concept of zero (0). It's because zero is the number for (empty set).

- We therefore use practical example:
- By asking children to identify groups of pupils with two heads in their classroom or group of pupils taller than the school block
- Children see that in each case the group has no members
- We then guide the children to realize that the number of members in each group is zero
- Guide children by asking them to identify a group of pupils with two heads in their class or someone who is taller than school flag pole. (Pupils note that it is impossible (event). It means 'no member' and 'no member' means 'nothing' which means 'zero'. Or



## **COUNTING OBJECTS IN A GROUP**

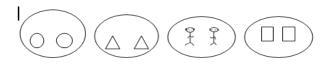
• To count objects in a group, the child move each objects in the group away from those not yet counted to a new position before he/she assigns a number name in the right order



- The last number name said gives the number of objects in the new position.
- He/she must be taught the number names conventional order of the number name i.e. one, two, three...

QU. Describe briefly how you would help a pupil to understand clearly the concept of the number "two".

• Show groups of various objects each containing two objects to the pupil



- In each case call out the number name "two"
- Next ask the pupil to show you two object e.g. two pencil two eye, two books etc.

### EXAMPLE

(a) Why would you teach a child how to count?

- So that the child can determine the number of object in a given group or tell how many object there are in group
- (b) What relevant skills should a child have to be able to count object in a group efficiently?
- know the number names in the conventional order
- Able to **co-ordinate the number names to the group of object.**
- use the last number name to describe the number of objects in a set.

c) Describe in detail the steps you would take a pupil in basic one through to enable him count the object in a given group correctly?

- Explain that when we count the objects in a group we determine how many object are in the group.

- pick **an object** in the given group and place it at **a new position** and **say the number name** corresponding to the number of objects **in the conventional order** beginning with one ....

That is if the object is at the new place, he / she say "one"

If two object at the new place he/she says "two"

-Continue this activity until all the objects at the **original place** have **been moved to the new place.** 

-The child uses the last number name mentioned to describe the number of object in the group that is the last number name tells how many objects there in the group.

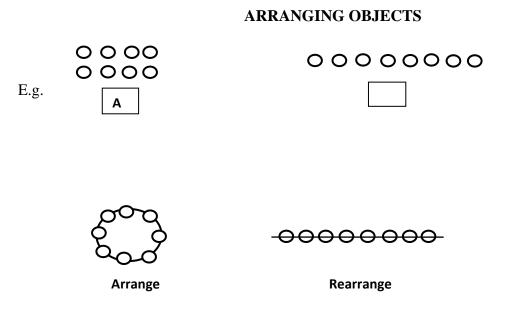
Q. What is counting?

It is assigning number names to a given group of objects in the right order or conventional order.

#### **CONSERVATION OF NUMBERS**

Materials: bottles tops, pebbles, maize grains, beads etc.

Conservation of numbers is the ability of a child to tell that, the number of objects in a group remains the same, when the objects in the group are re-arranged.



### Suitable activities to describe conservation of numbers

- Shows some grains/bottle tops in your palm to the pupils. As the pupil looks on, the teacher pours out these grains on the table or floor so that it is spread out.
- Ask pupil whether the grains on the table or floor are more than, less than or as many as the quantity of grains which were in your palm, without counting.
- The child is said to have conserved number if he/she says that there are as many grains on the table or floor as there were in the teachers palm without counting.

EXAMPLE:

(a)What do we mean when we say that a child has conserved number?

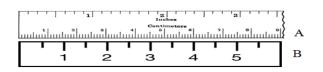
(b)Briefly describe how you would determine whether a child has conserved number or not.

### **CONSERVATION OF LENGTH**

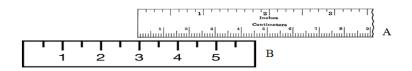
Is the ability of a child to tell that, the length of objects in a group remains the same when the objects in the group are re - arranged.

Activity (Materials, pens, pencils, rulers)

• Teacher shows objects of the same length to the pupils e.g. two rulers place together having the same length.



• Teacher moves one of the rulers forwards / backwards



Teacher asks the pupils whether ruler A or B, which one is longer or shorter

• The child is said to have conserved length if he/she says that, the two rulers are of the

same length if not, then that child cannot conserved length.

# **3** UNIT **3**: PLACE VALUE

**Place value** is the value of a digit in a number depending on its place or position in the number. Example: in the number 56, the '5' is 50 or 5 tens while '6' is 6 ones

In order to add a whole numbers with two three or more digits, we employed the use of the following base-ten teaching learning materials

- 1. Bundles of ten and single sticks
- 2. Dienes base ten blocks
- **3.** The abacus

## **3.1** Types of place value materials.

As groups of 10 become the dominant grouping activities, children will work with **proportional** and **non-proportional materials** use them to model numbers.

**Proportional materials** should be used first because the representative piece for 10 is actually 10 times the size of the piece that represents 1. Examples of proportional materials are:

- Single sticks, groups of 10 sticks, groups of 100 sticks, etc
- Dienes' base ten blocks units, longs, flats, blocks. (see Fig 1).

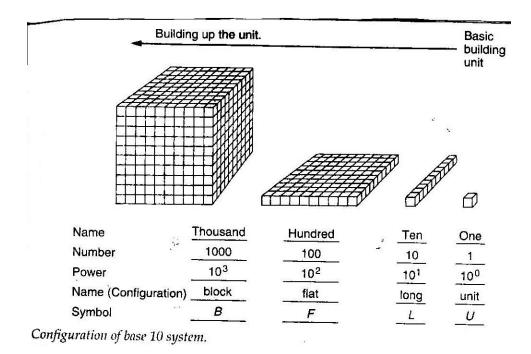


Fig.1 Source: Hatfield et al (2003)

In non-proportional materials, the same object is used to represent ones, tens, hundreds, etc.

For example, in the abacus the position of a bead on a rod determines its value rather than a visible "group of 10" or "group of 100".

Examples of non-proportional materials are:

- Abacus
- Place value mats/charts
- Money one pesewa coins, ten pesewas coins, one cedi coins, etc.

Both proportional and non-proportional materials can be used to represent or model numbers in the Hindu-Arabic numeration system. They can be used to explain the **concept of place value**, the understanding of which is useful in modeling 2-digit, 3-digit, etc. numbers. **Place value** means that the position of a digit in a number/represents the value of the digit in the number.

### **Dienes' Base Ten Blocks**

The base ten system begins with a single basic building unit, referred to as a **unit**. From there, each regrouping is done at ten. Dienes' base ten blocks are used in modelling numbers in the base ten system.

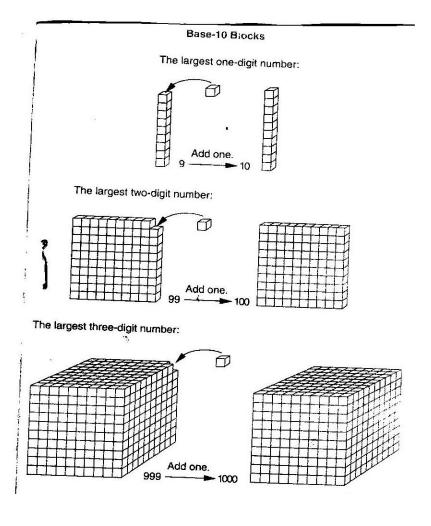


Fig. 2 Source: Reys et al (2004)

All units are single ones until the first regrouping. The configuration (shape) of the first regrouping looks like a long set of units fused together, hence the name long, representing 10. The numeral 10 means one set of ten.

The second type of regrouping occurs when there are ten longs (or ten groups of ten). The configuration looks like a flat with its ten longs fused together, hence the name **flat**. Ten times ten sets of the basic building unit can be seen in the **flat**, or one hundred pieces of the basic unit. The numeral 100 is used to express the number represented by a flat.

The third type of regrouping occurs when there a ten flats (or ten groups of hundreds). The configuration looks like a block with its ten flats fused together, hence the name block. Ten times a hundred sets of the basic building unit can be seen in the block, or one thousand pieces of the basic unit. The numeral expressing the number represented by a block is 1000. (see Fig. 1 for the configurations)

#### Developing and modelling two-digit, three-digit whole numbers

- Use Dienes' base ten materials to show two-digit numbers, for example, 26, by taking 2 longs and 6 units.
- Model three-digit whole numbers, for example, 134, using 1 flat, 3 longs and 4 units.
- \* Participants do Activity 1, Activity 2 and Activity 3.

#### Basic operations involving 2-digits, 3-digit, etc. whole numbers

### Addition

- Addition of 2- and 3-digit whole numbers using Dienes' base ten materials, without regrouping.
- Regrouping and renaming.

• Addition of 2- and 3-digit whole numbers using Dienes' base ten materials, with regrouping.

### Subtraction

• Subtraction involving 2-digit and 3-digit whole numbers using Dienes' base ten materials without regrouping.

Eg. 76 – 54; 349 – 235

• Subtraction involving 2-digit and 3-digits whole numbers using Dienes' base ten materials, with regrouping.

Eg. 62 – 48; 101 – 89

### Take-away subtraction with regrouping

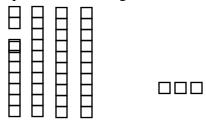
The first kind of interpretation for subtraction is the take-away approach. It is the easiest for a child to understand and the most common one that a child encounters in real-life situations, and it is simple to represent physically.

**Situation Problem:** Princess had 43 pesewas. She bought an eraser for 27 pesewas. How much money does she have left?

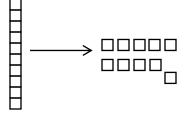
Materials: Dienes' base ten materials

### Procedure

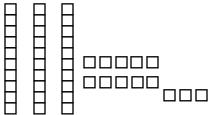
Step 1: Show 43 pesewas as 4 longs and 3 units (4 tens and 3 ones)



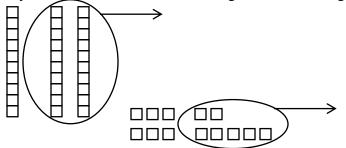
Step 2: Can't take 7 units from 3 units; so regroup (decompose) 1 long for 10 units.



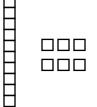
Step 3: Put the 10 units together with the 3 units. We now have 3 longs and 13 units.



Step 4: Now take away 7 units from 13 units and 2 longs from the 3 longs.



Thus, 1 long and 6 units are left; that is 1 ten and 6 ones or 16



Therefore 43 - 27 = 16

#### Teaching comparison subtraction with regrouping using Dienes' base ten materials

Another interpretation of subtraction is the comparison idea. Here a comparison is made between two sets.

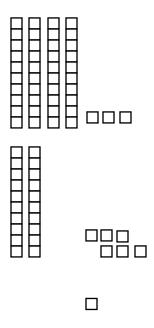
The questions on comparison involve ideas of how much more, how many more, how much older, and so on where the two sets are compared. Many children confuse the word "more" with the idea of addition. In the comparison interpretation, the modelling involves making both sets and determining the difference by one-to-one matching.

**Situational Problem**: Princess has 43 pesewas and Borenyi has 27 pesewas. How much more money has Princess than Borenyi?

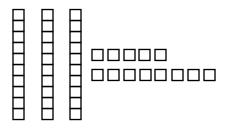
Materials: Dienes' base ten materials.

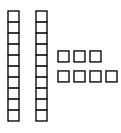
# Procedure

Step 1: Show 43 pesewas as 4 longs and 3 units (4 tens and 3 ones) and 27 pesewas as 2 longs and 7 units (2 tens and 7 ones)



Step 2: Comparing the units, we notice that 7 is greater than 3. So we regroup or exchange 1 long for 10 units and add them to the 3 units making 13 units. We now have 3 longs and 13 units and 2 longs and 7 units.





**Step 3:** We compare the 13 units to the 7 units and the 3 longs to the 2 longs and notice the difference as 1 long and 6 units, that is 1 ten and 6 ones or 16 units.

**Step 4:** We conclude, that 43 - 27 = 16

#### **Teaching Multiplication using Dienes' base ten materials**

Multiplication also has several interpretations; repeated addition, a rectangular array or row-by column, and a combination type. It is important not to limit the children's exposure to only one approach, or they will also be limited in their abilities to decide when a problem-solving situation calls for multiplication.

**Repeated Groups**: Generally, explorations begin with repeated addition to extend the approach most likely used by children when developing competencies for basic facts of multiplication. The repeated addition meaning of multiplication seems the easiest for pupils to understand and apply.

The language of repeated addition focuses on the number of groups and size of each group.

Interpretation of multiplication:

 $4 \times 5 = 4$  groups of 5 = 5 + 5 + 5 + 5

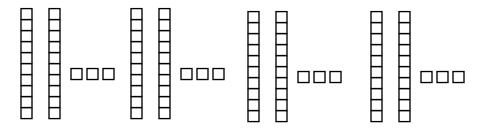
In the vertical form,

4 groups of 
$$5 = 5$$
  
 $\times 4$   
20

Note: This form places the 4 as the second number and still is read as 4 groups of 5.

Problem: Philemon sold 4 books of tickets. If each book contained 23 tickets. How many tickets did he sell?

The multiplication problem  $4 \times 23$  can be modelled with Dienes' base 10 materials as shown below:



 $4 \times 23 = 4$  groups of 23

Step 1: Represent 23 as 2 longs and 3 units.

Step 2: Form 4 groups of 2 longs and 3 units.

**Step 3:** Count the units = 12

# Step 4: Exchange 10 units for 1 long

 $\Rightarrow$  12 units = 1 long and 2 units

Step 5: Count the longs (9, i.e. 1 long added to 8 longs)

**Step 6:** Explain that 9 longs and 2 units represent the number 92

**Step 7:** Conclude that,  $4 \times 23 = 92$ .

Now consider  $11 \times 14$ 

Using Dienes' base ten materials, we proceed as follows:

Step 1: Represent 14 as 1 long and 4 units

Step 2: Form 11 groups 1 long and 4 units

Step 3: Count the units: 44 units

Step 4: Exchange 40 units for 4 longs.

 $\Rightarrow$  44 units = 4 longs and 4 units

Step 5: Count the longs (15, i.e. 4 longs added to 11 longs)

Step 6: Exchange 10 longs for 1 flat.

 $\Rightarrow$  15 longs and 4 units = 1 flat 5 longs and 4 units

Step 7: Explain that 1 flat, 5 longs and 4 units represent the number 154

**Step 8:** conclude that  $11 \times 14 = 154$ 

#### **Teaching Division using Dienes' base ten materials**

The division algorithm requires a knowledge of subtraction and multiplication algorithms, estimation skills, and an understanding of place value. In developing the algorithm, instruction should include the use of concrete models such as Dienes' base ten materials. In choosing the manipulative materials to model the algorithm, the actions on the materials, the language used to describe the actions, and the corresponding steps of the algorithm must be in total agreement. This is especially important in division where two different interpretations or approaches to the algorithm are possible.

**The two concepts of division are partitive division and quotitive** (or measurement) division. Both of these concepts can be modelled with materials.

**Partitive division** involves the process of taking a group of objects and giving them out or sharing (partitioning) them evenly into a specific number of groups until they are all gone, or none are left for another complete round. The objects are usually given out one at a time to those among whom they are shared until there are none left to give out to each of them again.

**Eg.** Huzeifa has 12 mangoes. He wants to give them out to his 4 friends so that everyone has an equal amount. How many mangoes will each friend get?

**Quotitive** (**Measurement**) **division** involves the process of taking a group of objects and separating (measuring) them into equal size groups of a specific size until are distributed.

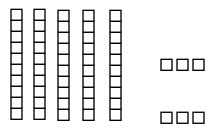
Eg. Ibtihaaj has 12 marbles and wants to give 3 to each friend. How many friends will get 3 marbles?

Let's solve the following division problems using Dienes' base ten materials:

**Problem**: There are 56 members in a Maths club. The president wants to form 4 equal teams. How many members will be in a team? The solution to the problem requires the use of the partitive approach.

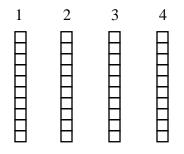
The problem:  $56 \div 4$ 

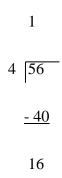
Step 1: Represent 56 as



Step 2: Distribute the 5 tens evenly to the 4 teams







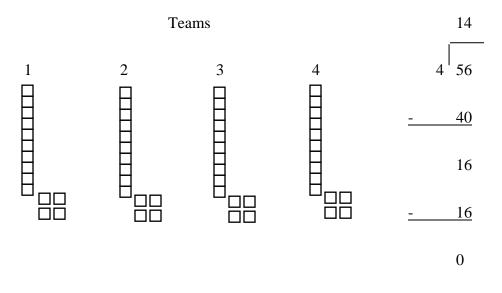
Record 1 in the tens column

Step 3: Break the one ten left into 10 ones and add them to the 6 ones. This gives 16 ones.



Step 4: Give 4 ones to each team.

### Record 4 in ones column



Each team has 14 members

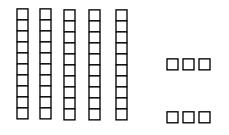
Thus  $56 \div 4 = 14$ 

**Problem**: There are 56 members in a Maths club. The president puts 4 members in a team. How many teams will there be?

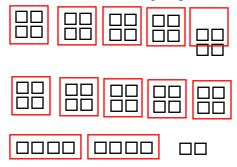
The solution to this problem requires the use of the quotitive approach.

The problem:  $56 \div 4$ 

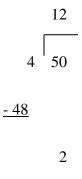
Step 1: Represent 56 as



Step 2: Break the 5 tens into 50 ones and group them in fours.



Step 3: Determine the number of groups of 4. There are 12 groups of 4 and 2 ones left. Record 1 ten in the tens column and 2 ones in the ones column.

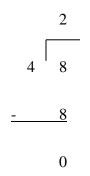


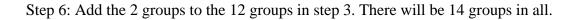
Step 4: Add the 2 ones left to the 6 ones to give 8 ones.

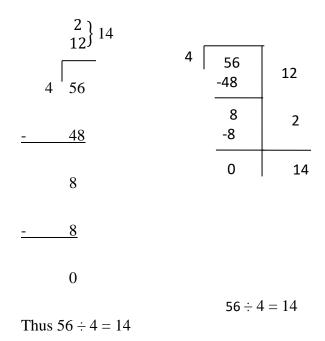
Step 5: Put the 8 ones into groups of 4.

|--|--|

There are 2 groups of 4.







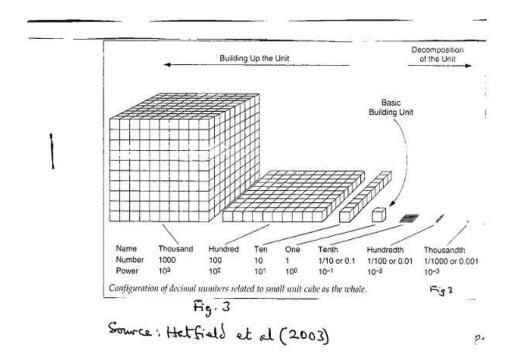
Notice that the results obtained using both the partitive and the quotitive approaches are the same, but the physical manipulation and the mathematical language to explain the steps are different.

• Participants do Activity 11 in groups.

# 4 UNIT 4: DECIMAL NUMBERS

Decimal numbers are another name for rational numbers in the base of ten numeration system.

The fractional parts of the base ten system are known as the decimal part of the system.



**Fig. 3** shows the build-up of the base ten whole number system. A new part has been added to the right. It illustrates the decomposition of the base ten system. By extending the place-value system into smaller segments of the basic building unit and keeping the same configuration of units, we obtain the rational numbers.

Just as the whole numbers are built up by groups of ten. So the fractional part of the basic building unit can be partitioned by groups of ten. Every time we move one place to the right, the

value of the digits is one-tenth as great. If the small unit cube is the whole, the configuration of the tenth is that of the flat or one-tenth of the basic building unit (fig. 3). The next regrouping is one-tenth of the small flat or one-hundredth of the basic building unit. It looks like a small long. The next regrouping is the one-tenth of the small long or one-thousandth of the basic building unit. It looks like a little block. We notice that the three new categories of base ten pieces fit into the same configuration as the whole number system.

However, the base ten pieces are only manufactured in units, longs, flats, and blocks. Therefore, modelling decimal number using base ten pieces become more complicated. The unit (tiny cube) cannot realistically be sub-divided easily into tenths and hundredths as fig. 3 illustrates. The pieces would be too small to handle. Since the central idea is maintaining the ten-to-one relationship, the basic building unit (one whole) could be represented by various pieces. If the whole is the block (large cube), the small cube (unit) would be one-thousandth, the long would be one-hundredth and the flat would be one-tenth. Thus, 6 tenth would be represented as 6 flats; and 2.3 would be represented as 2 blocks and 3 flats. Suppose the flat represents one whole, the value of a small cube (unit) will be one-hundredth, the long will be one-tenth and the block will be one ten. Then 3.42 will be represented as 3 flats, 4 longs and 2 small cubes (units).

#### Addition of decimal numbers

Finding sums of decimal numbers should be an extension of whole number addition.

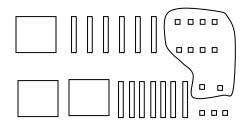
**<u>Problem</u>**: Kofi ran 1.68 kilometres, and Kwame ran 2.75 kilometres. How many kilometres did they ran in all?

**Solution**: Use a flat as the whole; then a long is one-tenth and a unit is one-hundredth. 1.68 is represented as 1 flat, 6 longs and 8 units. 2.75 is represented as 2 flats, 7 longs, and 5 units.

To find the sum 1.68

+ <u>2.75</u>

**Step 1:** Put 8 hundredths and 5 hundredths together to obtain 13 hundredths.



Step 2: Exchange 10 hundredths for 1 tenth and record the 3 hundredths left in the hundredths column.

1.68

+ 2.75

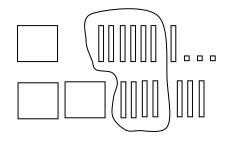


Step 3: Add 6 tenths, 7 tenths, and 1 tenth to obtain 14 tenths. Exchange 10 tenths for 1 whole and record the 4 tenths left in the tenths column 1

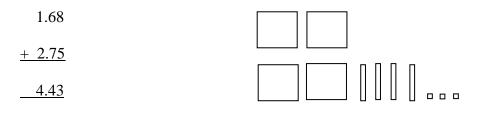
1.68

+2.75

.43



Step 4: Add 1, 2, and 1 to get 4 and record it in the ones column. (ONE)



Thus, Kofi and Kwame ran 4.43 kilometres in all.

\* Participants do Activity 12 in groups.

### Subtraction of decimal numbers

Subtraction of decimal numbers can be approached by using Dienes' base ten materils, like addition of decimal numbers.

**Problem:** Adisa bought 2.3 metres of ribbon. She used 1.7 metres for a design in her dress. How many metres of ribbon are left?

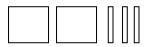
### Solution:

2.3 - 1.7

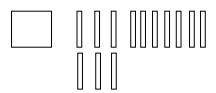
Let a flat represent 1 whole.

Then a long represents one-tenth.

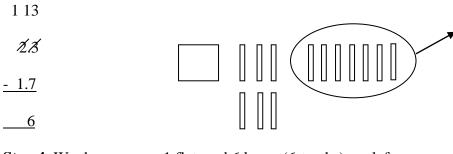
Step 1: Represent 2.3 as 2 flats (2) and 3 longs (3-tenth)



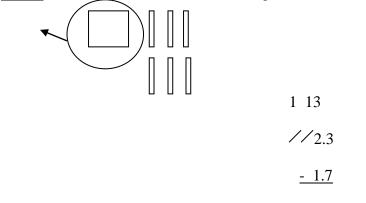
<u>Step 2</u>: Since we cannot remove 7 longs (7-tenths) from 3 longs (3-tenths), we exchange 1 flat (1) for 10 longs (10-tenths) and add them to the 3 longs (3-tenths) to make 13 longs (13-tenths).



**<u>Step 3</u>**: Remove 7 longs (7-tenths) from 13 longs (13-tenths) and we now have in all 1flat and 6 longs (6-tenths) left.



Step 4: We then remove 1 flat, and 6 longs (6-tenths) are left.

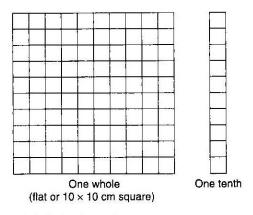


<u>0.6</u>

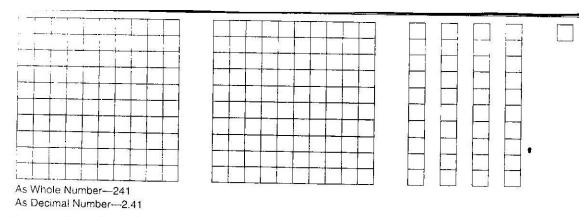
Thus, Adisa has 0.6 metre of ribbon left.

### **Multiplication of decimal numbers**

Multiplication of decimal numbers can be modelled using Dienes' base ten materials. The flat can be taken as the basic building unit representing one whole. The long represents tenths, and the small cube unit represents hundredths (see Fig. 4 and Fig. 5).



Models for decimals.

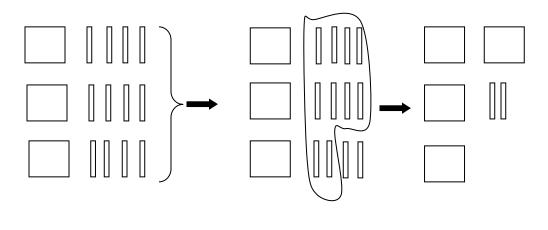


Interpreting base 10 models.

Ria 5

The following are examples of multiplication of decimal numbers using Dienes' base ten materials.

# i. 3 x 1.4



3 x 1.4

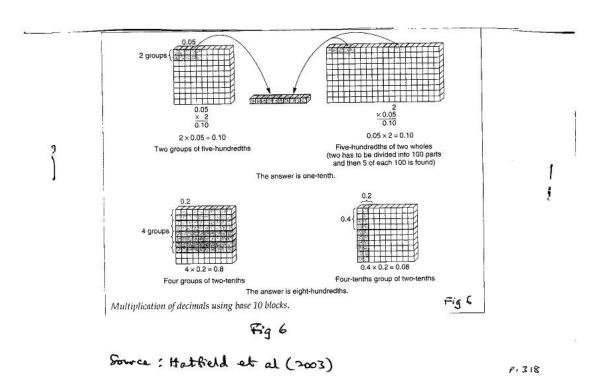
3 x 1.4 = 4.2

4.2

ii. 2 x 0.05

iii. 4 x 0.2

(See Fig. 6)



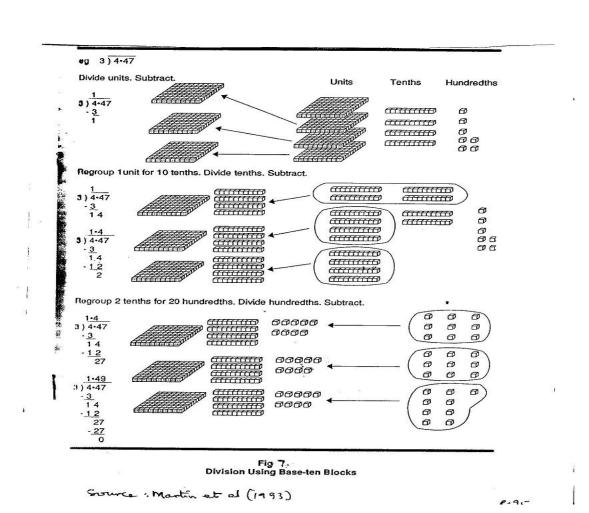
# 4.1 Division of decimal numbers.

Division of decimal numbers can also be modelled using Dienes' base ten materials. Here also, the flat, long and unit represent one whole, tenths and hundredths respectively.

The following is an examples of division of decimal numbers using Dienes' base ten material.

i.  $4.47 \div 3$ 

(See Fig. 7)



# 4.2 <u>ABACUS</u> (Chinese: Sua-pan; Japanese: Soroban)

An abacus is an example of non-proportional materials. In the abacus, the position of a bead on a rod determines its value. An abacus is an excellent device for extending children's understanding of place value to large numbers because it can show numbers that cannot be shown on other devices.

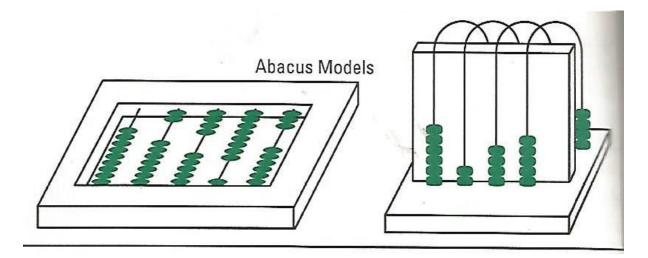


Fig. 8 (a)

Fig. 8 (b)

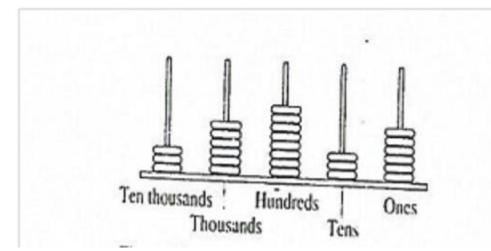


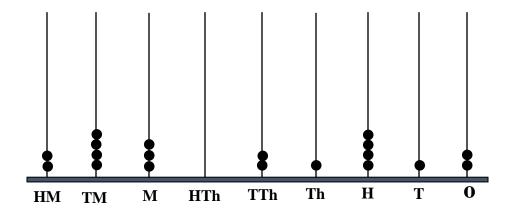
Fig. 8 (c)

Figs 8 (a), (b), and (c) show examples of abacus models. Columns are used from right to left. For base ten system, one bead in the column farthest to the right represent one, a bead in the second column represent ten, and a bead in the third column represent one hundred.

A first abacus should have only 3 or 4 rods. This abacus can be used to demonstrate to Primary class 3 or 4 pupils how it represents numbers. The teacher should point out that 1 bead on the tens rod has the same value (that is, it represents the same number) as ten beads on the ones rod; 1 bead on the hundreds rod represents the same number as ten beads on the tens rod.

The teacher should guide pupils to note that each rod represents a place-value position that has a value ten times as great as the position to its immediate right, regardless of where it is in the abacus. Later, with the help of the teacher, pupils will be able to reverse their thinking to conclude that each place value position also represents a value that is one-tenth of the value of 1 to its immediate left.

As their understanding of place value increases, pupils will generalize that the Hindu-Arabic numeration system has a place value scheme based on 10 and power of 10

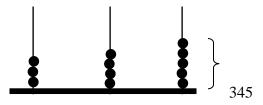


An abacus showing the number 243, 021, 412

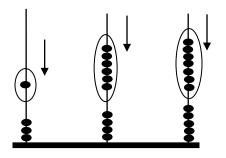
# 4.3 Addition of 2-digit or 3-digit whole numbers Addition using abacus

e.g 345 + 167

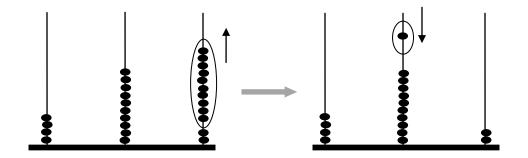
First, guide pupils to represent 345 on the abacus by placing three beads on the hundreds rod, four beads on the tens rod and five beads on the ones rod.



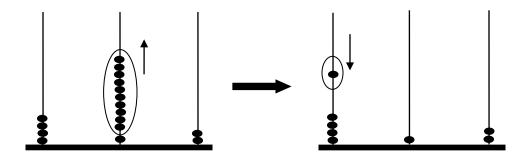
Let pupils add 167 by adding one bead on the hundreds rod, six beads on the tens rod and seven beads on the ones rod.



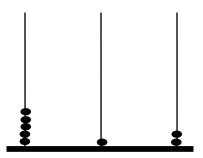
Explain to the pupils that if there are more than nine beads on the ones rod, they should remove a group of ten beads from this column and instead place one bead for it on the tens rod. From the twelve beads on the ones rod, therefore, the pupils remove ten and then put one bead on the tens rod.



Pupils note that there are two beads left on the ones rod and there are now eleven beads on the tens rod. They remove a group of ten beads from the tens rod and replace them with one bead on the hundreds rod.



Pupils notice that there are now five beads on the hundred rod, one bead on the tens rod, and two beads on the ones rod, representing 512



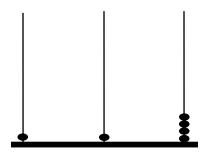
Hence 345 + 167 = 512

# 4.4 Subtraction of 2-digit and 3-digit whole numbers

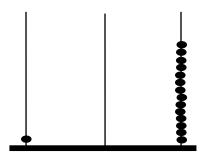
### Subtraction using abacus

E.g 114-39

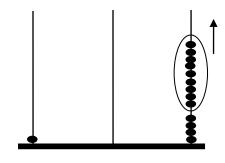
Guide pupils to represent the number 114 on an abacus



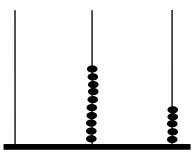
To subtract 39 from 114, pupils notice that they cannot remove nine beads from the four beads. Pupils are therefore guided to exchange one bead on the tens rod for ten beads and placed on the ones rod to give fourteen beads.



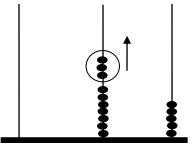
Pupils now remove nine beads from the fourteen on the ones rod, leaving five beads on that rod



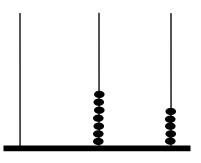
Pupils notice that there are now no bead on the tens rod. So to subtract 3 beads from the tens rod, they remove one bead from the hundreds rod and exchange it for ten beads and place them on the tens rod.



Pupils then remove three beads from the tens rod, leaving seven beads on it.



The pupils notice that there are now seven beads on the tens rod and five on the ones rod. The pupils notice that there are now seven beads on the tens rod and five on the ones rod.

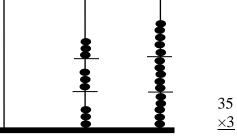


This represents the number 75. Hence 114 - 39 = 75

\*Participants do Activity 17 in groups

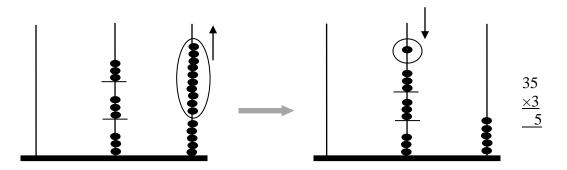
# 4.5 Multiplication using abacus

Pupils should be reminded that any time ten beads appear on a given rod, the beads must be exchanged for one bead on the rod immediately to the left. We demonstrate this are we solve  $3 \times 35$  on the abacus

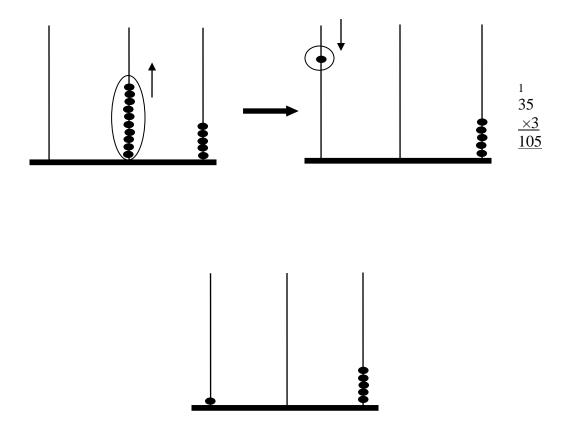


First, show  $3 \times 35$ 

The clips separate three representations of 35. Next we remove the holding clips on the ones rod and exchange ten beads ten beads on the ones rod for one bead on the tens rod. Five beads are left on the ones rod.



Similarly, we remove the holding clips on the tens rod and exchange ten beads on the tens rod for one bead on the hundreds rod. No beads are left on the tens rod.

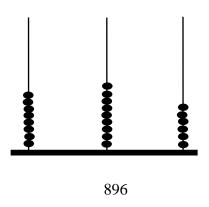


The last frame illustrates the abacus after all the exchanges have been made. Thus, we notice that  $35 \times 3 = 105$ 

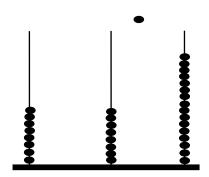
\*Participants do Activity 18 in groups

#### **Division of whole numbers using abacus**

An abacus can be used to show division of whole numbers. Let us consider the division problem  $896 \div 8$ . First we set the abacus up with 8 in the hundreds column, 9 in the tens column and 6 in the ones column.



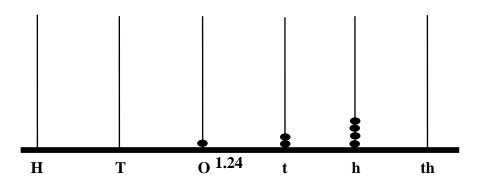
Next, we arrange the abacus into groups divisible by 8. The hundreds column has 8 beads which can be divided equally by 8. The tens column has 9 beads; so we leave 8 beads in the tens column and exchange 1 bead (1ten) from that column for ten beads (10ones) and place them in the ones column. We now have 8 hundreds, 8 tens and 16 ones.



We then divide each by 8 to get 1 hundred, 1 ten and 2 ones or 112. Thus,  $896 \div 8 = 112$ 

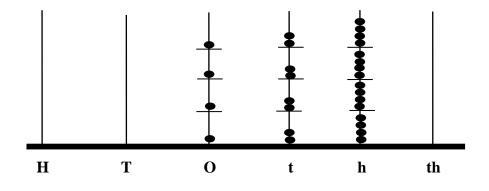
## **Decimal Abacus**

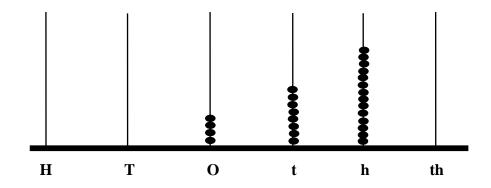
Multiply 1.24 by 4 using a decimal abacus



**Step 1**: Guide pupils to place one bead on the ones rod, two beads on the tenths rod, and four beads on the hundredths rod.

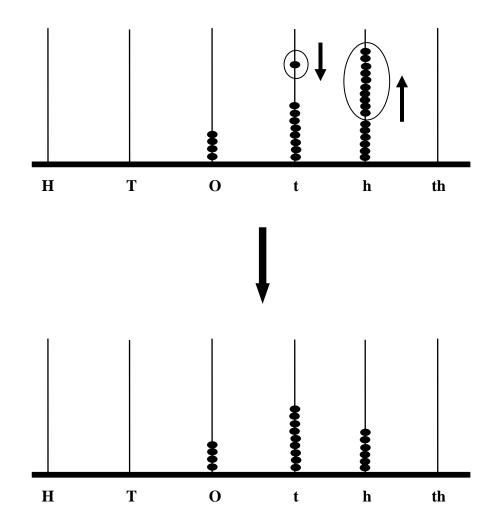
**Step 2**: Let pupils repeat step 1 on the same abacus three more times holding the beads with clips each time





Step 3: Help pupils put beads in each column together by removing the clips.

**Step4:** Where there are ten or more beads on a rod, pupils are assisted to exchange ten for one bead and place it on the rod to the left of it. Since there are 16 beads on the hundredths rod, pupils exchange ten for one bead and place it on the tenths rod



**Step 5:** Pupils notice that there are now four beads on the ones rod, nine beads on the tenths rod and six beads on the hundredths rod, representing 4.96

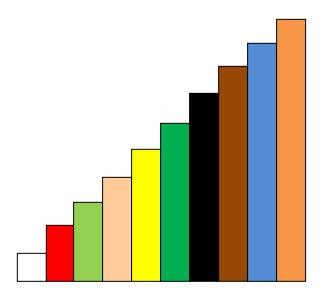
**Step 6:** Pupils conclude that  $1.24 \times 4 = 4.96$ 

\*Participants do Activity 21 in groups

#### **Cuisenaire Rods**

Cuisenaire rods are different coloured rods in graduated lengths from 1 centimetre to 10 centimetres. Each rod is associated with a particular number recognized by the colour and length of the rod. An arrangement of the rod from the shortest to th

e longest and their colours are shown as follows:



# 4.6 Using Cuisenaire rods to solve addition problem



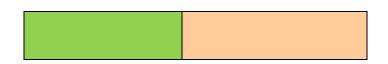
Cuisenaire rods are useful for developing addition concepts, for example 5 + 2 = 7

Cuisenaire rods can be helpful in showing different combinations of addends that are equivalent to a particular sum. For example, combinations of addends whose sum is 6 and the corresponding rod equivalents are shown below:



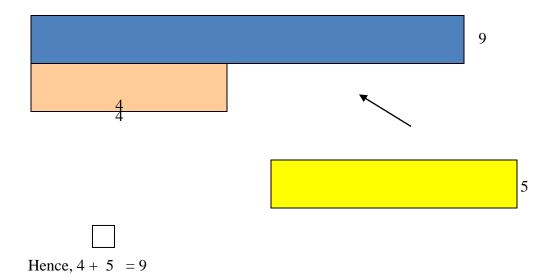
Commutative property of addition:





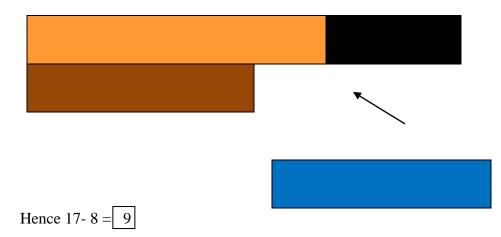
#### Using Cuisenaire rods to solve subtraction problem

Cuisenaire rods are useful for developing solutions to 'how many more' subtractions. For example, a subtraction problem such as  $4 + \boxed{\phantom{0}} = 9$  can be solved by the following procedures: The blue rod symbolizing 9 is placed on a table. Below it is placed the purple rod, which represents 4. The pupil's task is to find the rod that, when placed end to end with the purple rod will together be of the same length as the blue rod.



Also, subtraction using Cuisenaire rods can be performed by putting a train of rods along another train of rods so that they match at one end. Pupils will then find a rod or train of rods that fits along the remaining section of the longer. Pupils notice that the train of rods that fits the remaining length represents the difference. For example:

17 – 8 =



## Using Cuisenaire rods to solve multiplication problem

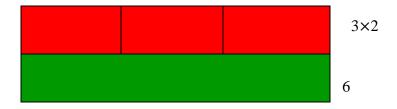
We can solve a multiplication problem for example,  $3 \times 2$  by finding the rod that is of the same length as three red rods placed end to end.

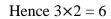
This is shown as follows:

 $3 \times 2$  can be represented as three red rods placed end to end.



We then find the rod that matches the length of the three red rods put together.





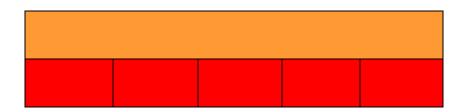
\*Participants do Activity 20(a)

## Using Cuisenaire rods to solve division problem

We can solve the division problem  $10 \div 2$ , for example, as follows:

Guide pupils to place red rods end to end along the orange rod and determine how many of the

red rods make the length of the orange rod.



Pupils notice that 5 red rods make the length of the orange rod.

Hence  $10 \div 2 = 5$ 

# 5 UNIT 5: WHAT IS ASSESSMENT FOR LEARNING (AFL)?

## ASSESSMENT FOR LEARNING INVOLVES MONITORING LEARNERS' PROGRESS AND ACHIEVEMENT TOWARDS CLEARLY DEFINED LEARNING GOALS.

"Assessment for Learning is any assessment for which the first priority in its design is to serve the purpose of promoting learners' learning. It thus differs from summative assessment (exams etc.) which is designed primarily to serve the purposes of accountability, or of ranking or of certifying competence. An assessment activity can help learning if it provides information to be used as feedback, by teachers and by their learners in assessing themselves, to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet learning needs." – (*Black et al., 2003*)

This differs to:

- assessment *of* learning (which is a summative process) aimed at evaluating the learner's academic achievement at a point in time, such as the end of a topic, term or academic year; and
- assessment *as* learning, where the assessment is a tool used to create a learning moment for example using peer or self-assessment to further the learner's learning and aid their personal goal setting.

# Why is it important?

Assessment for learning is a key pedagogical tool for:

- Establishing where the learners are in their learning
- Establishing where they are going
- Working out how to get them there

This process involves the learners, the teachers and classroom peers.

(William, 2009)

**Assessment for learning:** It is an approach used to monitor learner's progress and achievement. This occurs throughout the learning process. The teacher employs assessment for learning to seek and interpret evidence which serves as timely feedback to refine their teaching strategies and improve learners' performance. Learners become actively involved in the learning process and gain confidence in what they are expected to learn.

(NaCCA 2019 - Teacher resource nack)

# 5.1 What are the key elements of this approach?

The five key strategies to this approach are:

- 1. Clarifying, understanding, and sharing learning objectives
- 2. Questioning and engineering effective classroom discussions, tasks and activities that elicit evidence of learning
- 3. Providing feedback that moves learners forward
- 4. Activating learners as learning resources for one another
- 5. Activating learners as owners of their own learning

(Thompson & William, 2007)

The following are a sample of activities that you can try in your classroom. These can be adapted to be applied to all subjects and stages of education.

# 5.2 Shared Learning Objectives

Promote learner autonomy over their learning progression by sharing with them the learning objectives, and most importantly the success criteria.

#### • Learners write or ask questions

For example -

- About what they would like to know on a new topic;
- To ask the teacher or other learners in order to assess their learning;
- To demonstrate their learning/misconceptions/areas they would like to further explore.

#### • Lesson Target Setting

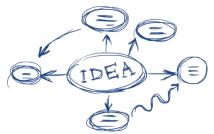
- Make the lesson more purposeful for learners by setting targets at the beginning about what you and the class are going to do;
- These can be referred to through the lesson and/or revisited in the plenary;
- Learners could then show how they have met targets in the plenary and/or set targets for next lesson.

#### • Making objectives clear

- Put lesson objectives on the board at the beginning of the lesson;
- Talk to learners about why they are studying what they are studying;
- Contextualise short-term objectives in long-term objectives and make real life application clear (e.g. understanding the nature of things in the environment – living and non-living will contribute to our wider understanding of the world around us) and;
- Check with learners that they are clear about the objectives of the lesson/unit/subject.

#### • Brainstorming

 Brainstorming is a technique used to determine what a learner may already know about a particular topic. Learners often feel free to participate because there is no criticism or judgment.

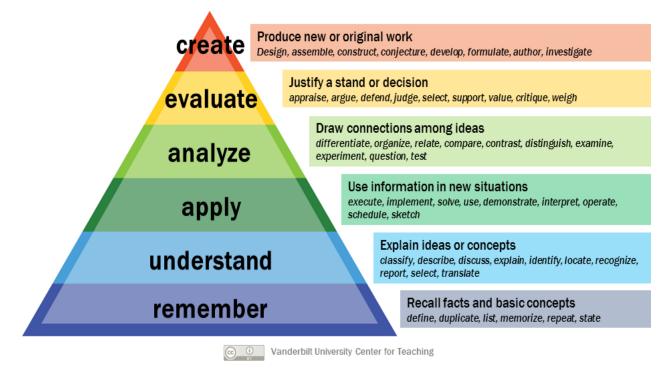


• Follow this on with clear objectives of what concepts need covering in the lesson (to consolidate and clarify understandings)

# 5.3 Questioning

#### • Bloom's Taxonomy

Bloom's Taxonomy provides an important framework for teachers to use to focus on higher order thinking skills. By providing a hierarchy of levels, this taxonomy can assist teachers in designing performance tasks, designing questions for discussing with learners, and providing feedback on learners' work.



- Questioning support sentences and starter questions
  - See Bloom's teacher planning kit worksheet on the next page

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# Source: CEBM (2016)

#### • Devising Questions

Devise questions that -

- Challenge common mistaken beliefs about a topic (misconceptions)
- Create conflict that requires discussion
- Explore ambiguity and encourage discussion and clarification
- Wait-time
  - Wait time allows learners time to think and therefore to produce answers. Also, not everyone in the class thinks at the same speed or in the same way waiting allows learners to build their thoughts and explore what has been asked.
  - $\circ$  2 types of wait time
    - Teacher speaks and then waits before taking learner responses.
    - Learner response ends and then teacher waits before responding. This gives the learner space to elaborate or continue – or for another learner to respond.

#### **5.4 Observations**

Teacher observations can be made in the course of delivery, during times of questioning and feedback, and when learners are engaged in activities, either alone or with peers or groups. Look out for the look of confusion, nod or spark of understanding etc. We observe to be responsive and adjust to keep the learning going or notice when it is time to stop or recap a concept.

#### Teach Collaboration

Peer assessment requires learners to act collaboratively. Indeed, AfL is a collaborative enterprise. Therefore, explicitly teach skills of collaboration. This process can be assisted by discussing collaboration with learners and making it visible as a part of the classroom. This can be achieved by encouraging:

- Group work in its many forms
- Pairs or peer to per work as a regular classroom practice or routine
- Building learners capacity to provide constructive feedback (see Section 2.4 on Feedback)

## • Tell your neighbour

- Learners 'tell their neighbour' as a means of articulating their thoughts.
- Ask a question, give thinking time and then ask learners to tell their neighbour their thoughts.
- This can either prepare whole class for 'hands down' questioning (where teacher asks randomly selected learner to contribute) or can precede a whole class discussion.

### • Think – Pair – Share

Give learners the opportunity to articulate their thinking before answering:

- Allow 30 seconds 1-minute silent thinking before any answers
- Ask learners to write some thoughts down before answering
- Ask learners to brainstorm in pairs first for 2-3 minutes
- Then get learners ready to talk about their own ideas or their group's ideas in a whole class discussion
- Think Pair Square
  - Think-Pair-Square is the same as Think-Pair-Share except that learners share their answers with another pair instead of the whole class.
- Debates
  - Debates enable the teacher to informally evaluate learners' oral work by assessing their oral presentation skills in terms of their ability to understand concepts and present them to others in an orderly fashion.

### 5.5 Feedback

Feedback is crucial in AfL to check help the learner know what they need to do to move their learning on. Effective feedback should:

- Be clear about what has achieved and what the learner still has to do;
- ▶ Be focused on the learning objectives and not the learner;
- Encourage a dialogue (where appropriate), so the learner can probe for clarification on next steps needed to progress their learning;
- > Be phrased so the learner can understand how he/she should respond and;
- > Be given at a time when the response will help the learner improve their learning.

# **Success Criteria**

It is important in the learning cycle that the learners and teacher are all aware of what will demonstrate that learning has taken place. This can be achieved by using the learning objectives and writing 'can do' statements that reflect this objective. Using Bloom's is very helpful here and this distinguishes the proficiency of the learner and can help make clear what the learner's next steps would be.

#### **INCREASING LEVEL OF BLOOM'S TAXONOMY**

B 2 .1 .2 .3 . 1	Low	Med	High
Describe a solid– solid mixture and explain how to separate the components	I can correctly identify and describe a solid- solid mixture	I can give an example of a solid- solid mixture	I can explain how to separate a solid-solid mixture

#### • Post-It / Slate/ Mini-whiteboard/ Rough-work book

Use post-it notes (or the other materials above) to evaluate learning. Groups, pairs or individuals can answer:

- Did I meet the success criteria?
- What should be done to improve next time?

Or:

- What have I learnt?
- What have I found easy?
- What have I found difficult?
- What do I want to know now?

### • $\mathbf{K} - \mathbf{W} - \mathbf{L}$

• At the beginning of a topic learners create a grid with three columns –

What They <u>K</u> now	What They <u>W</u> ant To Know	What They Have <u>L</u> earnt	

- They begin by brainstorming and filling in the first two columns and then return to the third at the end of the unit (or refer throughout).
- Variation extra column 'How Will I Learn'

#### Response Partners

- Paired or partnership oral marking. Learners invite a partner or a group to discuss or comment on their work. For it to be effective, learners should be aware of learning objectives and success criteria. They should also appreciate the role of a response partner – to offer positive and constructive feedback around the learning goals.
- Learners could be given prompt questions to ask the person who has done the work.

#### • Exemplar Work

- When setting learners a piece of work, show them examples that make it clear what it is they are being asked to do and what they need to do in order to meet the assessment criteria.
- Learners could mark exemplar work using the assessment criteria. This will help model what is being asked for and how it relates to the process of assessment.

#### • 2 stars and a wish

For peer assessment, ask learners to give two stars and a wish.

- $\circ$  Two stars = 2 things that are good about the piece of work
- $\circ$  A wish = something they can improve to make it even better

## • Traffic Lights

Use traffic lights as a visual means of showing understanding. Coloured card or paper could be used.



• Variation – Using smiley faces

Where coloured card is unavailable, simple face emojiscan be used to communicate learner's understanding.



### • Hand Signals

• Hand signals range from learners raising their hands to respond to a question posed by the teacher to a group "thumbs up/down" signal to determine learners "acknowledged" understanding of a concept or process.



When using traffic lights or hand signal techniques, it is important to ask a few follow up questions to check learners' actual level understanding. Learners who are confident can also be used to support or explain to others who are not as confident yet.

### • Comment-only marking

- Comment-only marking provides learners with a focus for progression instead of a reward or punishment for their ego (as a grade does). Comments could be made in books, in a table at the front of books, in a learning diary or journal. The latter are helpful for teacher and learner to track the progression of comments and see improvement.
- Comments should make it clear how the learner can improve. Plan activities and work with feedback in mind let the design assist the process.

### • Show and Tell

During teaching, you can use mini-whiteboards / slates / rough-work book so that every learner can write or draw their answer and show it to you (or their peers) immediately.
 Follow up with questioning to check for genuine understanding or to build upon answers especially in subjects like Mathematics where there is often one answer.

## Learning scenarios

Case Study 1

Consider these two lesson scenarios<sup>1</sup> and then discuss and answer the questions below:

### Scenario 1

When the teacher enters her B3 class, some of the learners are shouting at each other; some are talking quietly; some are moving around restlessly; and others are quietly waiting for the lesson to begin. She claps her hands and the learners take out their books. The teacher asks the learners what they read yesterday. When they tell her, she asks them to go to the next story, 'The Hare and the Hyena'.

One of the learners, who has been chosen to always be the reader in this class, stands up and begins to read this story. While the boy is reading, some learners are still trying to find their book, and others are still talking; many of them do not have a book to follow. Only a few learners are paying attention to the text and listening to the boy read. While this is happening, the teacher completes the attendance register and occasionally looks up and shouts "Hey, look at your books and follow. I am going to ask some questions. All of those

<sup>&</sup>lt;sup>1</sup>Adapted from: Changing teaching practices using curriculum differentiation to respond to students' diversity, UNESCO, (2004),

who do not answer the questions correctly will have extra homework."

When the text has been read aloud by the reader, the teacher asks a few questions about the text. Those who are listening and know the answers raise their hands and the teacher calls on them to give the answers. The teacher tells the learners to read the text again at home for homework, then the lesson ends.

#### Scenario 2

When the teacher enters her B3 class she spends a few minutes talking to the class, encouraging them to relax, interact, smile and laugh. The learners see she is carrying a book, 'The Hare and the Hyena' and the teacher holds the book up so all the learners can see it. Then the teacher introduces the book by asking questions about it to gain their interest. She asks questions at different levels for example:

'What colours are on the cover of this book?' 'What is on the cover of this book?' 'Who do you think are the main characters in this book?' 'What is the name of the book?' 'Who is the author?' What do you think the book is about?

She waits a moment after each question to give an opportunity for all the learners to raise their hands before choosing one to give the answer. Sometimes she asks a learner who hasn't raised their hand, if she thinks they will know the answer but are being lazy. The teacher goes on to ask other members of the class to explain what an author does, and then asks the learners to raise their hands if they know what a hare and a hyena are. She asks if any of the learners have seen these animals and what the animals did.

The teacher asks the learners what they think the hare and the hyena will do in the book. She then asks them to work in pairs to discuss what they think the story is about and how the story might begin and end. After a few minutes, the teacher asks one pair to give the results of their discussion. She asks if any other pairs think the same and they raise their hands. She then asks any of the remaining pairs for their thoughts. Finally, the teacher reads the first part of the story. As homework, she asks her learners to bring stories or information about the animals from their family, community members or elder peers for the next day. She tells them they will read and find out the end of the story tomorrow.

#### **Discuss and answer:**

- 1) Which of the two lessons do you think is most effective and why?
- 2) How does the teacher gain and maintain all learners' attention, participation and engagement?
- 3) What does the teacher do to accommodate all ability levels?
- 4) How does each teacher assess learning?
- 5) Can you find any missed opportunities in scenario 2 where the teacher could have used assessment for learning techniques?
- 6) How does the teacher use existing material and human resources in an interesting way?

# Case Study 2

Consider these two lesson scenarios and then discuss and answer the questions below:

The science teacher enters the B5 class and shouts for everyone to sit down and listen. She waits while some of the learners take out their books. The teacher tells the class they will learn about the respiratory system in humans. She asks the class to look at the relevant page in the textbook, and then asks one of the learners to read aloud while the others follow.

While the text is being read aloud, the teacher marks the exercise books from another class. Although few learners in the class pay any attention to the reading, when it is complete the teacher asks them to re-read the text aloud together. She then tells them to copy the diagram of the human respiratory system into their exercise books.

The teacher then continues marking as before until the end of the lesson when she asks for a show of hands of the learners that have completed the diagram. Five learners raise their hands and the teacher says 'good' and tells the others to complete it for homework.

Scenario 1

Scenario 2

Discuss and answer:

- 1) Which of the two lessons do you think is most effective and why?
- 2) How does the teacher gain and maintain all learners' attention, participation and engagement?
- 3) What does the teacher do to accommodate all ability levels?
- 4) How does each teacher assess learning?
- 5) Can you find any missed opportunities in scenario 2 where the teacher could have used assessment for learning techniques?
- 6) How does the teacher use existing material and human resources in an interesting way?

# 5.6 How assessment in mathematics can be done?

In 1992, the national council of teachers of mathematics (NCTM) undertook the development of a report on assessment to complement its earlier curriculum and evaluation standards for school mathematics. Their policy is summarized on the table below

Toward	Away from
Assessing student's full mathematical power	Assessing only student's knowledge of specific
	facts and isolated skills
Comparing student's performance with	Comparing student's performance with that of
established criteria	other students
Giving support to teachers and credence to	Design" teacher-proof" assessment systems
their informed judgment	
Making the assessment process public	Making the assessment process secrete,
participatory and dynamic	exclusive and fixed
Proving students multiple opportunities to	Restricting students to a single way for
demonstrate their full mathematical	demonstrating mathematical power.
power	

#### Table 4.3Major shifts in assessment practice

The central problem of changing the nature of assessment in mathematics must be faced in the design of actual mathematics assessments that reflect these guidelines.

Assessment must focus on the important grade –level appropriate in mathematics. Since assessment can only be a sample from all that is learnt, it must be sampled as effectively as possible- by concentrating on the most important and useful mathematics taught and learned at grade level, as defined by NCTM.

✤ Assessment are worthwhile learning activities – not digressions from learning for the student, assessment is a tool that helps further the understanding of important mathematical ideas. For the teacher, assessment is student work that informs and augments instruction. Worthwhile assessment is not something students and teachers "stop and do" but a way to further what they are already doing.

#### 5.7 ASSESSMENT STRATEGIES

There are several types of assessment strategies and among such strategies include:

Concept test: a concept test is a technique used often in a lecture setting. The instructor presents one or more questions during class along with several possible (plausible) answers. Students in the class indicate which answer they think is correct. This could be done by a show of hands, for example, if many of the students do not give the correct answer, students are given a short time in lecture to try to persuade their neighbors that got the correct answer. The question is asked a second time to gauge class mastery. The technique can be essentially useful in large classes.

**Knowledge survey**: knowledge survey consists of series of acquisition that cover the full content of a course. The surveys evaluate student learning and content mastery at all levels: from basic knowledge to comprehensive, through higher level of thinking. Knowledge surveys can serve as both formative and summative assessment tool. They help students to learn, help faculty to improve classroom, and aid departments and programs as they explore new curricula and pedagogies.

## 5.8 THE ASSESSMENT PRINCIPLE

Assessment should support the learning of important mathematics and furnish both students and teachers with useful information.

When assessment is **an integral part of mathematics instruction.** It contributes significantly to all students' mathematical learning. When assessment is discussed in connection with standards, the focus is sometimes on using test to certify students' attainment, but there is other important purpose of assessment. Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions. Rather, it should be an integral part of

instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done to students: rather it should also be done for students, to guide and enhance their learning.

The assessment standard for school (NCTM, 1995) presented six standards about exemplary mathematics assessment. They addressed how assessment should-

- Reflect the mathematics that students should know and be able to do;
- Enhance mathematics learning;
- Promote equity
- Be an open process;
- Promote valid inference;
- Be a coherent process
- Assessment should enhance students learning, the assertion that basement should enhance student's learning may be surprising. After all, if assessment ascertain what students have learned and are able to do, how can it also have positive consequences for learning? Research indicates that making assessment is an integral part of classroom practice is associated with improved students learning.

Source of the same as the same as the activities used in instruction should be included. When teachers use assessment technique as observations, conversations, and interviews with students or interactive journals, students are likely to learn through the process of articulating their ideas and answering the teacher's question.

**Feedback from assessment task can also help students in setting goals**, assuming responsibility for their own learning and becoming more independent learners. For example, scoring guides or rubrics can help teachers analyze and describe student's response to complex tasks and determine student's level of proficiency. They can also help students understand the characteristics of complete and correct responses. Similarly, classroom discussions in which students present and evaluate different approaches to solving complex problems can prune their sense of the difference between an excellent response and one that is mediocre.

#### Assessment is a valuable tool for making instructional decisions

✤ To ensure deep, high- quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than a interruption. Such assessment also provides the information the teacher need to make appropriate instructional decisions. In addition to formal assessment such as tests and quizzes, teachers should be continually gathering information about their student's progress through formal mean, such as asking questions during the course of a lesson, conducting interviews with individual students and giving writing prompts.

When teachers have useful information about what students are learning, they can support their student's progress toward significant mathematical goals. The instructional decisions made by teachers- such as how and when to review prerequisite material, how to revisit a difficult concept, or how to adapt tasks for students who are struggling or for those who need enrichment-are based on inferences about what students know and what they need to learn. Assessment is a primary of the evidence on which these inferences are based and the decisions that teachers make will be only as good as that evidence.

Assessment should reflect the mathematics that all students need to know and be able to do. It should focus on student's understanding as well as their procedural skills. Teachers need to have a clear sense of what is to taught and learn. Assessment should be aligned with their instructional goals. By providing information about student's individual and collective progress toward the goals, assessment can help ensure that everyone moves productively in the right direction.

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#### The teacher as a planner of instruction

The very nature of instructional planning has changed because of the emphases on testing. Teachers typically and closely follow the state content standards of their individual states planning lessons and units. State content standards are the documents to which the individual state tests are aligned. Many principals require teachers to write the exact standard and correlating desired learning outcomes on their daily plans. In this way, the teacher can demonstrate that each learner outcomes was taught and re-taught on certain days prior testing.

Teachers must determine how best to teach students of various ability levels and backgrounds in a way that best ensures success for all students. Since the standards, desired learning outcomes(objectives) and assessment are to a great extent, predetermined, it is important that teachers understand the characteristics of the students and how to structure the teaching-learning environment to be best help them meet their individual educational potentials. Many points must be considered when preparing for high caliber instruction. Among the most important are questions concerning

- 1. The need of student
- 2. Objectives
- 3. Strategies
- 4. Resources
- 5. Evaluative criteria

Once these questions are addressed, the teacher's role in the classroom during the teaching process itself needs to be analyzed.

Teachers must be aware of their student's ability to perform academic task and the conditions under which they learn best (their learning style of performance). This is critical information to have when planning instructional adaptation to match students learning needs. As a group, students with high aptitudes need less direct instruction and time to complete learning tasks than those whose aptitudes are lower. The students of differing abilities often use different strategies when learning to solve problems, even same problem.

# 5.9 Scheme of Learning and Lesson Plan

To be able to teach mathematics effectively and even any other subject, there is the need for the teacher to prepare and plan well in advance before the actual lesson delivery.

The teacher has to;

- □ Prepare a scheme of Learning
- Divide the list you have prepared into three (3) to depict three terms.
- □ Prepare the termly scheme of Learning using the format.

# 5.10 WHAT IS SCHEME OF LEARNING?

• This is the termly plan prepared by the teacher indicating the strands and sub-strands that are to be covered in the term.

### Columns in a scheme of learning

• The columns in a scheme of learning include; week, strand, sub-strand, content standards, indicators and resources.

Week	strand	Sub-strand	Content	Indicators	Resources
			standard		

- Week refers to the order of the week in a term .e.g., 1, 2, 3, etc.
- Strands are the broad areas/sections of the mathematics content to be studied.
- Sub-strands are the topics within each strand under which the content is organised.
- **Content standard** refers to the pre-determined level of knowledge,skill and/or attitude that a learner attains by a set stage of education.

• **Indicator** is a clear outcome or milestone that learners have toexhibit in each year to meet the content standard expectation. The indicators represent the minimum expected standard in a year.

**Resources** are the TLMs & any item that will be used by learners and teachers to improve teaching & learning.

Week	Strand	Sub-Strand	Content standards	Indicators	Resources
1 & 2	Numbers	Counting representation & Cardinality	B1.1.1.1	B1.1.1.1.1 - 3	Counters, (e.g. bundle and loose straws) base ten cut-outs – squares and longs/strip,
3&4		Counting representation & Cardinality operations patterns	B1.1.1.1 B1.1.1.2 B1.1.2.1	B1.1.1.1.4-5 B1.1.1.2.1 B1.1.2.1.1	
5,6&7	Algebra	Operations Patterns	B1.1.1.2 B1.2.1.1	B1.1.2.1.1 B1.2.1.1.1	Counters, patterns made from manila cards

## 5.11 Steps to consider in preparing a scheme of learning in mathematics

When preparing the termly scheme of learning in mathematics, the teacher should take cognizance of the following steps:

- Examine the entire mathematics curriculum of the year you want to prepare on.
- List all the strands and sub-Strands in the mathematics curriculum of the year in question
- ✤ Assess the weighting/contents to teach each sub-Strands in relation to the teacher's handbook, pupils textbook and the time allocated for each week.

- Divide the list you have prepared into three (3) to depict three terms in a year, in doing this classification
- the teacher should consider :
  - ✤ The total number of functional weeks for each term
  - The seasons and suitable periods for some topics, e.g. improving teaching and learning material with guinea corn stalk after the rains.
  - The factor of integration (i.e. similar topics in other subjects) The sequence of the topics.

## LESSON PLANNING

- □ It is a detailed step by step plan describing what is to be taught, how it is to be taught and the evaluation process.
- This is the detailed plan of activities that the teacher or whoever takes up the lesson in the absence of the original teacher will undertake in order to achieve his or her learning outcome(s).
- □ The expanded scheme of learning can also be prepared lesson notes of the lesson plan.
- Below is the format for the preparation of the expanded scheme of work (LESSON PLAN).

#### **Template for Planning Daily Lessons**

Date:	Period:		Subject
Time:			Strand:
Class:	Class size:		Sub-Strand:
Content Standard		Indicator:	Lesson 1 of 2
Performance Indicator:			Core Competencies and Subject
			Specific Practices:
Key words:	-		
Phase/Duration	Learner activities		Resources
Phase1: Starter (preparing the			
brain for learning)			
10 minutes			
Phase 2: Main (new			
learning including			
assessment)			
40 minutes			
Phase 3: Plenary/ Reflections			
(Learner and teacher)			
10 minutes			

DATE: it is the actual date on which the lesson will be delivered.

- TIME : The duration is a specification of the length of time allocated to the lesson. E.g. 30minutes, 60 minutes etc.
- CLASS: the specific class that the materials are meant for should be clearly stated. If there is more than one stream of class, the streams should be clearly stated.
- SUBJECT: the particular subject for which the lesson note is prepared is to be specified .e.g. Mathematics.
- > Strands are the broad areas/sections of the mathematics content to be studied.
- Sub-strands are the topics within each strand under which the content is organised.
- Content standard refers to the pre-determined level of knowledge, skill and/or attitude that a learner attains by a set stage of education.

- Indicator is a clear outcome or milestone that learners have to exhibit in each year to meet the content standard expectation. The indicators represent the minimum expected standard in a year.
- LESSON
- PERFORMANCE INDICATOR is a type of key Performance indicator which is a measurable value that demonstrates how effectively a company is achieving
- > Are critical indicators of progress towards an intended results.

**CORE COMPETENCIES** The core competencies for studies describe a body of skills that teachers in Mathematics at all levels should seek to develop in their learners. They are ways in which teachers and learners in Mathematics engage with the subject matter as they learn the subject. The competencies presented here describe a connected body of core skills that are acquired throughout the processes of teaching and learning.

- ✓ Phase 1: Starters (preparing the brain for learning)
- ✓ Phase 2: Main (new learning including assessment)
- ✓ Phase 3: Plenary/Reflections (Learner and teacher)

The main (new learning including assessment) phase in a lesson plan should be devoted to having *all* learners:

- explore the new learning areas for the day
- work in pairs or groups to carry out differentiated tasks
- work with resources or tools to carry out differentiated tasks
- share and discuss their results and strategies

develop relevant core competencies (i.e. problem solving, critical thinking, communication, digital literacy, collaboration etc.)

• <u>Assessment</u>, in each lesson, time should be set aside for learners to work independently or collaboratively on problems. (teacher should move around, look at and check learners' work and provide remediation where appropriate.)

#### The Plenary/Reflections phase -

to reflect, recap on and consolidate the learning. Introduce ideas that will be visited in the next lesson, forming a bridge for continuous learning. Ideally, it:

- ✓ occurs at the end of a lesson but can also be used at other points in the lesson if appropriate
- ✓ Brings the whole group together to participate
- ✓ Enables both teacher and learners to check on learning so far and to identify any misconceptions that need to be corrected
- ✓ Directs learners to the next phase of learning
- $\checkmark$  Helps learners to understand not only what they have learned, but also how they learned it
- ✓ Some useful plenary techniques:
- ✓ Post-it notes for learners to collect 3 things they have learnt.
- ✓ Using a flip chart or whiteboard/chalkboard to collect group learning achievements
- ✓ Showing 5, 4, 3, 2, 1 fingers to demonstrate success in learning progress on a five-scale point. For example, 5 fingers mean 'I really got it', 4 means 'mostly got it', 3 means 'got some of it', 2 means 'got little of it' and 1 means 'didn't get it'.
- ✓ . Sit in a hot seat and make 3 points (of things learned in the lesson) as a key character that would be an expert in the lesson outcome, hand on to another class member who has to make 2 points, then down to 1.
- ✓ Note: Always finish on time so that you don't miss out the plenary in your lesson plan. It should be very clear to any observer that learners have made progress in learning and can demonstrate it. Where insufficient progress has been made it should be clear what the plan of action will be to address this next lesson. One of the most common criticisms is that teachers don't use assessment outcomes to inform future planning. Make it clear that you can see what each and every learner has learnt and what the next steps are to secure progress.

SINGLE, 60 MINUTE CLASS		TWO BACK-TO- BACK 30- MINUTE CLASSES (EACH CLASS FOLLOWS SAME ROUTINE)		
Durati	Activity	Durat	Activity	
on		ion		
10 min	A starter should	5	A starter should stimulate	
	stimulate curiosity	min	curiosity and open	
	and open		mindedness and prepare	
	mindedness and		the brain for learning.	
	prepare the brain		These can be random	
	for learning. These		and/or linked to the	
	can be random		content standard.	
	and/or linked to the		E.g. Random Mental	
	content standard.		activities (fast paced	

Durat ion	Activity	Duratio n	Activity
IOII			
40	Activities to explore new	20 min	Activities to explore new
min	learning content for day		instructional content for
	(including at least 10		day (including at least 5
	minutes where pupils do		minutes where pupils do
	problems or exercises		problems or exercises
	alone or collaboratively, in		alone or collaboratively, in
	their exercise books and		their exercise books and
	teacher move around to		teacher circulates to
	monitor and check work)		monitor and check work)
10min	Plenary/Reflections	5 min	Plenary/Reflections
8	Reflect, recap on and		Reflect, recap on and
	consolidate the learning		consolidate the learning
	that has happened in the		that has happened in the
	davia lasson		darya lagaan

# 6 Unit 6: SHAPE, SPACE AND MEASUREMENT

# Money:

The use of money is so important in our lives that pupils should be able to identify the currency and the various denominations in use.

# **Recognition of currency**: Notes and coins.

- Make available the currency notes and coins in circulation and ask children to examine and identify them in their denominations.
- Give each person the opportunity to (co-operative learning/group work) take each of the notes and coins and mention its value.

**Note**: At the lower primary, initially, coins and notes up to Gh¢1.00 may be given to them to examine and identify them and ask pupils to mention the value an item it can buy. For example you can buy bread, kenkey etc. and later they examine the currency notes of larger denomination.

# **Buying and selling**

- We need to provide children with valuable experience with money.
- Teacher can create play shops in the classroom where the children play the roles of shopkeepers, shop attendants and customers (Role play).
- Allow pupils to practice buying and selling articles.
- The one buying decides on the combinations of coins and notes to use in paying for their articles. And customers give out bigger denomination of currency and agree with the shopkeeper on the amount of change.

### 6.1 Time

**Time: It** is the period between two events. For examples, periods between the rising and the setting of the sun, period between when one is born and one dies, etc.

Time may be measured objectively or subjectively.

Objectively; use of devices such as stop-watches, stop clock digital watch and clock.

Subjectively: It involves one's own judgment from his experience of the passing of time. For examples40 minutes waiting for a bus might be seen longer than 40minutes of playing football.



# 6.2 Awareness of passage of time (idea of time)

How the child will be able to tell the time some events occur in the day.

For children to understand the concept of passage of time, we need to expose them to experiences in which they become aware of when an event activity takes place.

#### Examples;

(1) Activities of breakfast, sunrise, time they go to school, washing your face, pasting their teeth are for morning.

(2) Activities like time they close from school, lunch, playing football, seeing shadow just under them are for afternoon

- (3) Activities like sunset, sleeping, supper, lighted lamp on the table, are for evening (night)
  - Children are given pictures of events or activities.
  - Children indicate or tell when given activities are carried out on a particular day-morning, afternoon or evening.

Note: Events like yesterday, today and tomorrow

- Days of week
- $\blacktriangleright$  4 weeks in a month
- $\succ$  12 months in a year
- $\triangleright$

**Q.** Using two suitable examples briefly explain the idea of the passage of time to children in an early childhood class.

# Awareness of duration of event

- ➤ Guide the child to identify event which takes a short time and those which take a long time.
- Guide the child to compare two event and determine which one will take a short time or long time.

E.g

• Take breakfast, walking to the staff common room

Take a book from the office, japing 3 times Emptying water from containers with different sizes of holes.

Note: we can use timing devices and use them as an arbitrary unity.

- We fill a funnel with sand. The amount of time the sand to empty becomes an arbitrary units of time.
- Another timer we can develop is a candle on which several equally spaced marks have been made. The amount of time for the candle to burn from one mark to another becomes an arbitrary unit.
- We can also construct a time by making holes in the bottom of a can and fill it with water. The amount of time for the can to empty the water becomes arbitrary units.

# 6.3 Reading the clock

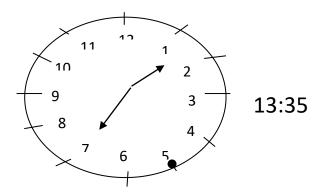
Note: Although digital clocks are easier to read, they do not allow children to see the position of the displayed time relative to the next hour or to the times that before or after it.

To simplify the task of learning to tell time, one approach is to begin with only an hour-face clock. Using only an hour hand allows children to read the time as soon as they are able to recognize the numerals 1 to 12.

• We introduce the minute hand when children tell the time by the hour.

The child must be able to count by 5 to 60 to tell the time to the nearest 5 minutes.

- Write down six skills a child need to develop to be able to tell the time from a regular(analogue) clock.
  - a. Identifying the hour and the minutes hand.
  - b. Identifying the divisions of the clock face
  - c. Telling the time by hour, noting the minute hand is on 12 while the hour is on the number indicating the hour.
  - d. Identifying the hour that a time is after or before. E.g. it is after 4 0'clock



## **Measurement of Capacity**

Use ECE note (estimation and add this one)

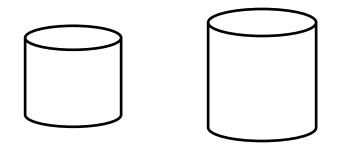
Measurement of Capacity: Comparisons between two containers to determine which of them hold more quantity.

**Direct Comparison**: materials: empty milo and milk tins, plastic drinking cups and plastic containers we can find our environment.

In direct comparison of capacity, first allow the pupils to estimate first (guess which container can hold more water and sand when given two containers.

• Give a variety of containers to children or sand and pour it into the other to see which

holds more quantity. E.g.



## **Indirect** Comparison

Let pupils estimate the number of feeding bottle that will fill each jar and verify

• Indirect comparison when two containers cannot be compared directly, may be two then introduce the measurement of capacity using arbitrary units to measure capacities bottles as arbitrary unit(olonka), small cups, spoons, bottles as arbitrary units to measure capacities of large containers.

**Note**: The liter is introduced as the unit of most liquid measure. We introduce 'L' for liter and encourage them to record measurement of capacities using this symbol.

# 6.4 Measurement of Mass

### Mass –Comparing Heaviness/Lightness of objects

• Guide the child to compare the weight of objects using the expressions heavy and light

**NB:** The child at this level cannot use beam balance.

- Guide the child to weigh the objects in his/her palms to determine which one is heavier or lighter.
- Guide the child to note that, the palm that is lowered is where the heavier object is and the palm ta the top is where the lighter objects is.

Weight refers to a measure of the force of gravity acting on an object. Mass on the other hand, is the amount of matter in an object.

### **Direct Comparison:**

• Guide the pupils to discover that small stone is heavier than a big bag full of cotton wool by weighing in their palms.

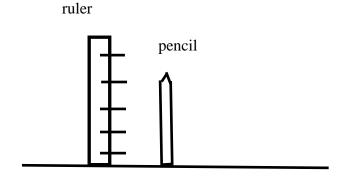
### Indirect comparison:

Note: Children may also use the see-saw to compare their masses with other members of their class and order themselves according to their masses. We may guide children to use a simple balance to determine which of two objects is heavier. The basic unit for the measurement of mass is kilogram(kg)

### Measurement of Length

Use the note in ECE book length direct/indirect longer and this note comparing lengths words **Direct Comparison** – the objects to be compare are brought together side by side on the same flat base surface and compare their lengths/heights with reference to common base to determine which one is longer than, shorter than or as the other. For example, one may compare the lengths of a pencil and a ruler on the teacher's table.

[comparing the lengths of a ruler and a pencil]



The ruler is **longer** than the pencil and the pencil is **shorter** than the ruler.

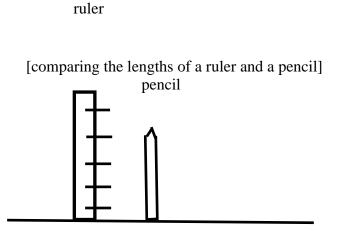
**Indirect Comparison:** is comparing of lengths of two objects which cannot be brought together on a common flat surface to compare their lengths, because of that we make use of an arbitrary units. E.g. Comparing the length of the teachers table and the length of the classroom window. Arbitrary unit can be use: hand span, pencil, pen.

#### The meter as a Standard Unit of Measure

Assuming we try to measure the length of the teachers table, two pupils to do it separately with their hand span. [one get 13- hand- spans and the other get 14 hand spans, the answer give use idea of how long the table is]. The problem is which answer is correct? This leads us to the use of standard unit of length. Therefore, we introduce the metre as a conventional standard unit.

a) Describe how you would introduce primary school pupils to measurement of length up to include the introduction of the meter as the standard unit for measurement of length.

- First I will introduce pupils to direct comparison of length where pupils bring objects together on the same flat base and compare their length or height and identify which is longer or taller.
- After that, I will introduce pupils to indirect comparison and measurement of as follow:
- Guide pupils to identify lengths of objects which cannot be brought together to measure.
- Ask pupils to use different arbitrary units to measure the lengths.
- Pupils notice that they obtain different results for the length using different arbitrary units.
- This gives rise to the need to use a standard unit of measure.
- Introduce the meter as the conventional standard unit for measurement of length.
- b) Explain each of the following terms as used in the measurement of length:
  - a. i) Direct Comparison
    - ii) Indirect Comparison
  - b. Describe an activity in which you would involve primary class 2 pupils in the measurement of length using each of the following
    - i. Direct comparison
    - ii. Indirect comparison
- a. i) Direct comparison in the measurement of length is comparing the lengths of objects by bring them together with their bases on the same flat surface and determine which is the longer or longest.
- ii) indirect comparison in the measurement of length of objects by using some units of measure (arbitrary units) such as pencils, pens, erasers etc.
- the unit of measure is matched with the length to be measured and the number of time that unit covers the entire length is determined from each of the lengths being compared and the longer or longest determined.
- b. i) I will guide pupils to bring a pen and a ruler together with their bases on the same flat surface. (teacher table)
- Pupils compare the length or heights of the objects to identify which is longer or taller (longer or longest)



The ruler is longer than the pencil and the pencil is shorter than the ruler.

- ii) Pupils are guided to identify lengths of objects which cannot be brought together to measure.For example, one may the length of the teacher's table and the length of the window in the classroom.
- I will guide the pupils to use arbitrary units (hand span, pencil, pen) to measure the lengths to be measured.
- Guide the pupils to determine the number of arbitrary units which cover each of the lengths beings measured and determine the longer or longest.
- Pupils realize that the length of the table is 10 hand spans and that of the window is 7 hand spans
- Therefore, the length of the table is longer than the length of the window.

## **Measurement of Area**

Surfaces of Objects – ask pupils to rub their hand over surfaces like the tops of tables and chairs, a globe, a rubber ball and so on and determine intuitively or by sight which of the two surfaces is larger.

A measure of the amount of "surface" an objects possesses is called area. Length is a linear measure and area is a two dimensional measure

### Estimating and measuring surfaces with arbitrary units.

Using arbitrary units (objects) to determine the area of surface.

Materials: bottles top, exercise books, playing cards etc.

• Guide the child to arrange objects on a given surface to find the number of objects that can cover the surface.

Act: Finding the area of the surface of the teacher's tables.

Materials: exercise books

- First allow the child to guess the number exercise books that can be arrange to covers the table.
- Guide the child to arrange the exercise books on the surface of the tables.
- Guide the child to verify by counting or finding out the number of exercise books that can covers the surface of the teacher table to represent the area of the table eg. 10 exercise books. Therefore the area is 10.

Act: Finding the area of the surface of a note book/ exercise books.

Materials: playing cards

Procedure: the same steps as the steps above

### **.Direct Comparison (Area)**

• Guide pupils to place one shape on top of another and tell which of the two surface is smaller or larger.

Activities: Let pupils cut the tops of various tins for example, empty milo, milk or cerelac tins and compare their surface by placing one on the other. Or a child may say that the top of the teachers table is larger than the cover of her exercise book.

#### **Indirect Comparison**

Note: Here we may not be able to compare surfaces by moving the objects to place one on top of the other. For example, one may compare the surface of the chalkboard to the surface of the classroom floor.

In such cases, arbitrary measurement units such as postage stamps, playing cards, books, empty match boxes, postcards, cut – out squares, rectangles may be used to compare their areas.

### Perimeter of plane Shapes

Perimeter of any plane shape is the total distance or lengths of the shape.

Q1. How will you guide pupils to find the perimeter of a given plane shape?

- I will guide the pupils to measure all the perimeter of a given plane shape.
- After which I will ask pupils to add up or sum all the measurement of sides/taken.

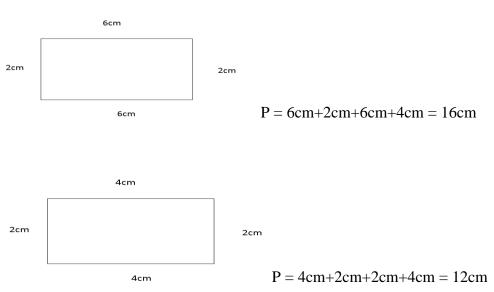
### Perimeter of rectangle and circle

Q. How would you guide pupils to find the perimeter of a rectangle.

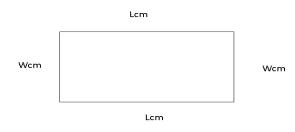
• Guide the pupils to measure the lengths of all the sides of the rectangle and record the measurement.

• Ask pupils to find the sum of the measures, which will give the perimeter of the rectangle.





In finding the relation between perimeter and length and width of a rectangle, ask pupils to draw a rectangle and measure it lengths to obtain Lcm and width as Wcm as shown below



Perimeter, P, of the rectangle

- =L+W+L+Wcm
- = L + L + W + Wcm
- = 2L + 2W cm

$$= 2(L+W)$$

Therefore, the perimeter of a rectangle is obtained by finding twice the sum of the measure of its length and its width.

### Perimeter of a Circle (Circumference)

- Guide the pupils to make a collection of circular objects such as empty milo tins, tomato tins, geisha tins and so on.
- Working in pairs, ask pupils to find the measurement of the circumferences of the objects in cm with the aid of a string or strip of paper or a tape measure.
- Ask pupils to find the objects between hard card or books or hard pieces of wood and measure the distance between. This will give the diameter of the circular object being measured. Again measure in cm.
- Guide pupils to draw a table to record their results.

Circular objects	Length of	Length of diameter (d)	Ratio (c ÷d)
	circumference (c)		
Milk tin			
Milo tin			
Tomato tin			
Geisha tin			

- Guide pupils to divide the circumference of each circular objects by its diameter. Allow pupils to examine the results obtain and compare them.
- By considering the average results together, it about 3.14 discover that the ratio  $\frac{c}{d}$  is the same or for all circle; meaning that C is proportional to "d". the constant of proportionality then is the irrational number denoted by  $\pi$ .

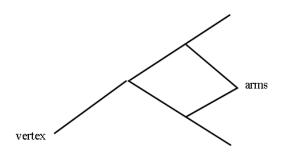
From the above activity you will realize that the ratio  $\frac{c}{d} = \pi$  or  $c\pi d$  (By multiplying both side by d)

Now show pupils that in a circle since there is relation 2r = d by substituting this in the above: we have

 $C = \pi \times 2r = 2\pi r.$ 

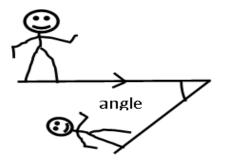
### **ANGLES**

An angle; It is the amount of turning made about a point.

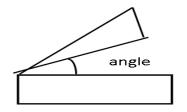


# 6.5 Activities of Explaining Angles to JHS Pupils

1. Walking on a straight line and suddenly change direction



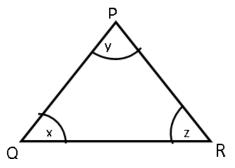
2. Opening of a book, window



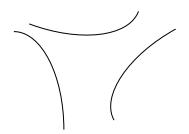
- 3. Swinging of the arms
- 4. The corner of the classroom

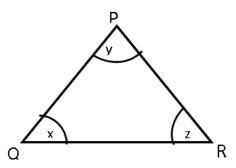
**Ques:** Describe how you will guide a JHS pupil to discover that the sum of interior angles of a triangle is  $180^{\circ}$ 

**Ans:** I will guide the pupils to make a cutout shape triangle PQR and label the inferior angles x, y, and z.

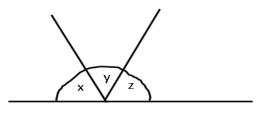


• I will guide the pupils to cut – off the 3 interior angle





I will guide the pupils to arrange the angles on a straight line (edge)



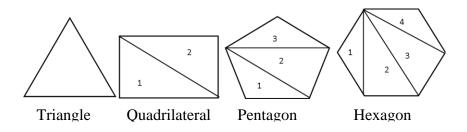
- I will guide the pupils to note that, since the sum of angles or straight line is 180<sup>0</sup>, it implies that, the sum of interior angles of a triangle is 180<sup>0</sup>
- Therefore,  $x + y + z = 180^{\circ}$

### **POLYGONS**

It is a simple closed figure bounded by straight edge

**Ques:** Describe in detail an activity which you will use to JHS pupils to find out that, the sum of interior angles of a polygon with n – sides is given by  $(n-2) \times 180^{\circ}$ 

**Ans:** I will guide the pupil to make cut – out shape of polygons and split them into triangles by drawing the diagonals from one of the vertex to all other verities.



- I will explain to the pupils that; interior angles of a triangle add up to  $180^{\circ}$ .
- I will guide the pupils to find the interior angles of the sum of each polygon by counting the number of triangles and multiply by 180<sup>o</sup> as shown in the table below.

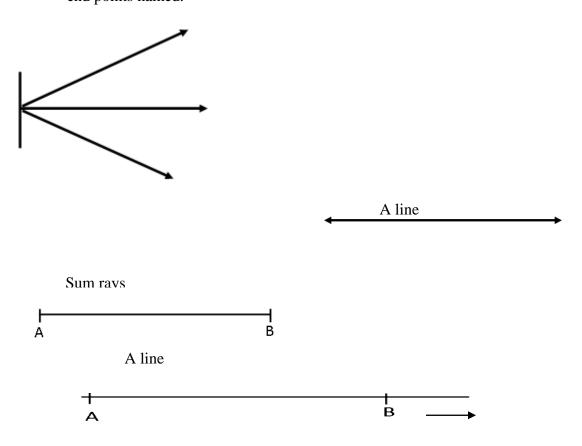
Polygon	Number of sides	Number of triangles	Sum of interior angles
Triangle	3	1	$1 \times 180^{\circ} = 180^{\circ}$
Quadrilateral	4	2	$2 \times 180^{\circ} = 360^{\circ}$
Pentagon	5	3	$3 \times 180^{\circ} = 540^{\circ}$
hexagon	6	4	$4 \times 180 = 720^{\circ}$
•	•	•	•
•	•	•	•
•	•	•	•
	n	n – 2	$(n-2) \times 180^{\circ}$

- I will ask the pupils to observe the number of sides and number a triangle for each polygon and look for a pattern.
- Pupils notice that, if you subtract "2" from the number of sides, you will get the number of triangles for each polygon.
- Therefore, the sum of interior of angles of a polygon with n sides is  $(n-2) \times 180^{\circ}$ .

# 6.6 Line Segments and Angles

Line Segment: tracing out the edge of a book or ruler we have a line segment.

- A line has no end points.
- A ray is a line with one end point named while a line segment is a line which has two end points named.



a ray from point A through point B

Q. Describe how you would guide primary pupils to find two differences between triangular prism and a triangular pyramid.

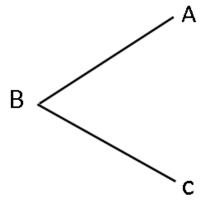
- Guide the pupils to find out that:
- i. A triangular prism has <u>uniform cross sections</u> which are triangular, whereas the triangular pyramid has non-uniform cross- sections which are also triangular.

- ii. A triangular prism has five (5) faces while a triangular pyramid has four (4)
- iii. The faces a triangular prism except those with uniform cross-section are rectangles, whereas all the faces of the triangular pyramid are triangles.
- iv. The faces of a triangular pyramid except the bases meet at a <u>point</u> (an apex); whereas those of the triangular prism do not.
- v. A triangular pyramid has four vertices while a triangular prism has 6 vertices.
- vi. A triangular pyramid has 6 edges while a triangular prism has 9.

## How to measure an angle using a protractor

Note: The instrument used to measure an angle is called a **protractor**.

Q. Describe briefly how you would guide pupils in Junior High School Form 1 to measure the angle ABC drawn below using a protractor



- Guide pupils to place the protractor in such a way that, the <u>base line of the protractor( $0^0 180^0$  line</u>) lie on one arm / ray of the angle.
- Also, let the <u>Centre of the protractor</u> (i.e. meeting <u>point of 90<sup>0</sup>line and the base line</u>) coincide with the vertex B of the angle.

- Guide pupils to read the <u>angle from the base line 0<sup>0</sup></u> to the <u>second ray / arm of the angle</u> and find the angle on the protractor which coincides with the line (extend the ray / arm where necessary)
- The angle measure  $48^{\circ}$ .

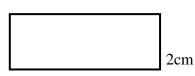
**Q**. Explain how you will lead upper primary pupils to find out for themselves three differences between a square and a rectangle.

- Lead pupils to fold the rectangular sheets of paper and square sheet of paper a long diagonals to find out that the diagonals of the squares are lines of symmetry, but those of rectangles are note
  - i. Pupils fold rectangular sheets of paper to determine that the square has 4 lines of symmetry but the rectangle has 2
  - ii. Pupils measure the sides of the square and rectangular shapes and note that all the sides of a square are equal in length, whereas in the rectangle only the pairs of the opposite sides are equation
- iii. Let pupils use the corner of the square to measure the intersection of the diagonals and determine that the diagonals of the squares bisect each other at  $90^{\circ}$  whereas the rectangles do not.

## Area of a rectangle

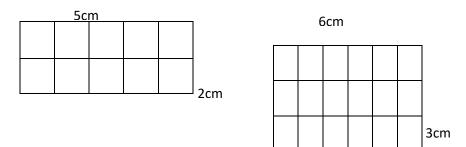
TLM: Using square centimeter cut outs. (1cm ×1cm)

- Lead pupils to find the area of the follow rectangles using  $(1 \text{cm} \times 1 \text{cm})$  square pieces.
- i. 6cm ii.

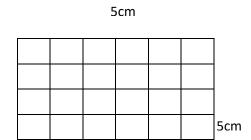


5cm

- ii. 6cm 4cm
- Guide pupils to arrange the pieces of  $1 \text{ cm} \times 1 \text{ cm}$  square piece on the various rectangles.



Area =  $10 \text{cm}^2$ 





- Guide pupils to count the number of square centimetres that occupy each of the rectangles, We have 18 squares, 10 squares and 24 squares respectively in i), ii), iii)

 $Area = 18 cm^2$ 

- Ask pupils to record what they notice as far as the relationship between the length and breathe as well as corresponding areas in all the cases.

Rectangle	Length (L)	Breadth (B)	Product of length	Area
			and breadth $(\mathbf{L} \times \mathbf{B})$	
Rectangle (i)	6cm	3cm	$6 \times 3 = 18 \mathrm{cm}^2$	18cm <sup>2</sup>
Rectangle (ii)	5cm	2cm	$5 \times 2 = 10 \mathrm{cm}^2$	10cm <sup>2</sup>
Rectangle (iii)	бст	4cm	$6 \times 4 = 24\mathbf{cm}^2$	24cm <sup>2</sup>

- Pupils will observe that, column 4 and 5 have the same values. This implies that area of a rectangle = product of length and breadth
- Therefore Area of rectangle =  $L \times B$ .

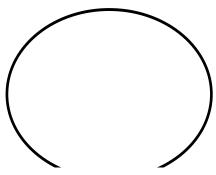
Note: Use a similarly approach to guide pupils to discover that area of a square =  $L \times L = L^2$ 

- **Q**. Describe the steps that you would go through with your pupils in primary class 4 to introduce them to the measurement of area, including the introduction of standard unit "the square centimeter" (cm<sup>2</sup>)
  - Lead pupils through direct Comparison leading to awareness of differences and the use of words bigger, smaller and larger to describe these differences
  - Guide pupils to use arbitrary unit such as stamp, playing cards, shapes to determine the space covered.
  - Guide pupils to estimate and measure the area of the Teachers table using arbitrary unit such as stamp, playing cards, and exercise books.
  - Pupils notice that in each case different results were obtained, for instance 20 stamps,
    15 playing cards and 10 exercise books recorded as the number of each arbitrary unit use for the area of the table.
  - Teacher then introduce the need for standard unit of measure (the square centimeter) due to the difference in the result obtained.

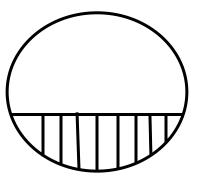
# Area of a Circle ( $A = \pi r^2$ )

**Qu**. How would you guide JHS pupils to discover that the area of a circle is  $r^2$ .

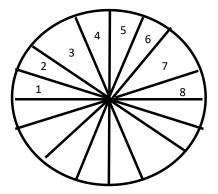
- Guide pupils to draw a circle, with diameter of at least 20cm, onto a piece of a card or paper



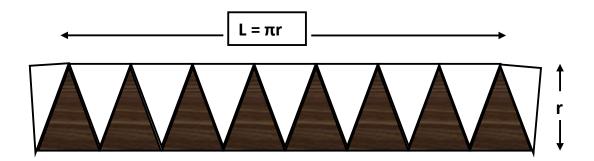
- Guide pupils to cut this out, fold it into two halve and colour one half.



- Guide pupils to fold again to form quarter and again and again to form eights and then sixteenths.



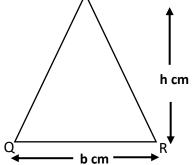
- Guide pupils to cut along the fold lines to obtain 8 coloured sectors and 8 uncoloured sectors. Guide pupils to cut one of the uncoloured
- Guide pupils to rearrange the pieces head to tail as show:



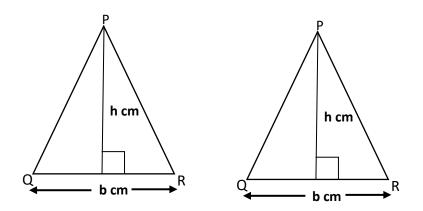
- Guide pupils to realize that, the shape is approximately a rectangle with length  $\pi r$  and breath r
- Therefore Area =  $\pi \mathbf{r} \times \mathbf{r} = \pi r^2$
- Explain to pupils that since the rectangle was obtained by rearranging the sectors of the circle, the area of the circle is the same as the area of the rectangle.
- Therefore area of the circle,  $A = \pi r^2$

### Area of a triangle

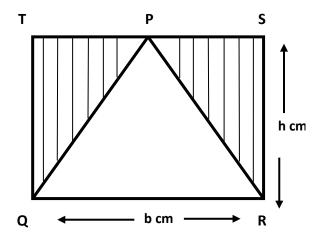
**Q**. Describe how you would guide JHS pupils to discover for themselves that, the area of the triangle PQR is  $\frac{1}{2} \times b \times h$  cm<sup>2</sup>, where b cm is the length of the base and h cm, the height of the triangle.



- Guide pupils to make <u>two identical triangular paper cut – outs</u> of the triangle with base length b cm and height h cm.



- Guide pupils to <u>cut up one of the triangular cutouts</u> into two triangles along the altitude and arrange the three pieces as shown below to obtain a rectangle.

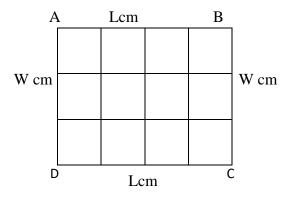


- Pupils find out that, the <u>area of rectangle</u>  $QRST = b \times h$ .
- Pupils notice that the rectangle QRST is made up of the two identical triangles.

- Pupils note, therefore, that: the area of one of the triangles is

A = 
$$\frac{1}{2}$$
 of the area of the rectangle QRST  
=  $\frac{1}{2}$  bh cm<sup>2</sup>

- Qu. A pupil finds it difficult to distinguish between the perimeter and the area of a plane shape. How would you help her out? Give examples to support your explanation.
  - Explain to the pupils that, perimeter is the <u>measure of the total distance around</u> theplane shape, while area is the <u>measure of the total surface of the plane shape</u>.
     For example, for the rectangle ABCD



The perimeter is L+W+L+W=2(L+W)cm and the area is  $Lcm \times Wcm = LW cm^2$ 

Q. Using at least four examples, describe how you would help to determine whether or not rectangles having the same area 36cm<sup>2</sup> also have the same perimeter.

- Lead pupils through examples of the rectangles having the same area:

Length = 36 cm, width = 1 cm

Length = 9 cm, width = 4 cm

Length = 18 cm, width = 2 cm

Length = 12 cm, width = 3 cm.

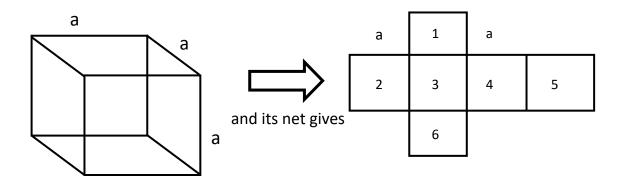
- Guide pupils to record their results in the table below

Length (L)	Width (W)	Perimeter (P)	Area (A)
36cm	1cm	74cm	36cm <sup>2</sup>
9cm	4cm	26cm	36cm <sup>2</sup>
12cm	18cm	40cm	36cm <sup>2</sup>
12cm	3cm	30cm	36cm <sup>2</sup>

- From the table, guide pupils to conclude that rectangle having the same area do not have the same perimeter.

have the same permeter.

Total surface area of solids (cubes and cuboids)



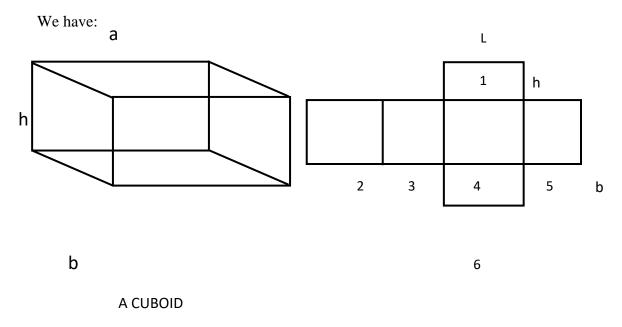
- In finding the total surface are of a cube, ask pupils how many faces can you find in a cube?
- Pupils response b. Explain to pupils that, the shape of each face of a cube is a square.
- Pupils find area of a square as  $a \times a = a^2$

- Let pupils add up all the area of the squares of a cube to obtain  $a^2 + a^2 + a^$
- Therefore we concluded that, a cube with side 'a' has a total surface area of  $6a^2$ .

# 6.7 Area and Volume of 3D Shapes

### SURFACE AREA OF A CUBOID

Note: This is found in a similar way to the a cube.



NET OF CUBOID

Note: Repeat the first steps of finding the surface area of a cube.

- Let pupils tell you the formula for area of a rectangle =  $L \times B$
- So Area of face  $1 = \text{Length} \times \text{height} = \text{Lh}$

Face  $2 = \text{Length} \times \text{breath} = \text{Lb}$ 

Face  $3 = breath \times height = bh$ 

Face  $4 = \text{Length} \times \text{breadth} = \text{Lb}$ 

Face  $5 = breath \times height = bh$ Face  $6 = length \times height = Lb$ 

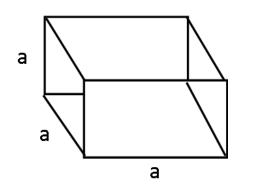
- Guide pupils to sum all the faces as:
  - = Lh + Lb + bh + Lb + bh + Lb
  - = 2 Lh + 2 Lb + 2 bh
  - = 2 (Lh + Lb + bh)
- We conclude that, the total surface area of a cuboid is 2Lh + 2 Lb + 2 bh, where L, b and h are length, breadth and height respectively.

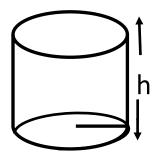
# 6.8 Volumes of Cuboids and Cubes

Volume is the amount of space an object of Solid takes up. In other words, it is the measure of the amount of space a solid shape can occupy. Volume is usually measured in cubic units.

A cuboid has all its six faces being rectangular. Also it has three dimensions namely, length, breath and height. Volume of a cuboid, can be find by asking pupils to find the product of the length, breadth and height.  $V = L \times B \times H = LBH$ 

Volume of a cube: Is the product of the lengths of the sides that gives.  $a \times a \times a = a^3$ 





## Volume of a Closed Cylinder

This is a cylinder with circular base radius 'r' and height 'h'. To find its volume we first find the area of the circular base and then multiply it by the height.

The volume of a cylinder = base area  $\times$  height

 $= \pi r^2 \times \mathbf{h}$  $= \pi r^2 \mathbf{h}.$ 

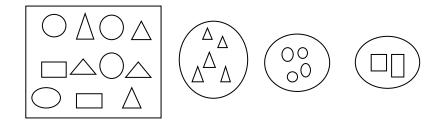
# 6.9 TEACHING GEOMETRIC SHAPES

Identifying and Naming Plane Shape or shapes e.g. rectangle, circle, star, kite triangle, semi-circle

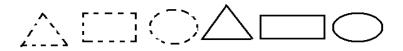
(moon)

-Guide the child to identify the plane shape and their name using cut-out shapes or shape draw on cards

-Guide the child to sort and group objects according to shapes of the same kind. e.g.



- Guide the child to identify and pick and call the name from the collection.
- Guide the child to trace or complete the shapes by drawing



• Guide the child to identify difference in shapes

**Qu.** Describe an activity in which you engage kindergarten pupils to enable them identify the following rectangle, circle, triangle.

• Show the pupils cut-outs of the given shapes.



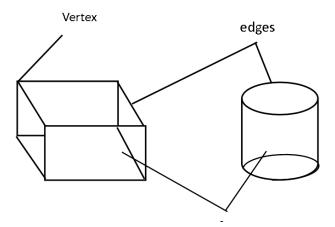
- Assist pupils to learn their names
- Give out the cut-out shapes (to groups or individuals) to examine carefully
- Pupils take turns to pick out from a collection a shape and name it.
- Pupils identify or pick a named shape from a collection of shape.
- Pupils identify a named shape from shapes drawing on a card or chalkboard.
- Pupils call out the name of a shape and another pupil draws it or traces it from a cutout.
- Pupils identify or point to a named shape in a read object.

### Solid Shapes

Note: It is important to allow pupils to use manipulative materials to explore concepts in both two

- and three - dimensional space and discover relationships.

There are two main groups of solid shapes. They are prisms and pyramids.



- Face is the flat side (plane) of a solid
- An edge is the meeting line of two faces. It can be straight or curve.
- Vertex is the point where two or more edges meet.
- Cuboids include cubes, cylinders.
- Prisms: they are made up of triangular prisms, pentagonal prisms and even cubes, cuboids and cylinder.

Pyramids: made up of square pyramids, triangular pyramids, rectangular pyramids, pentagonal pyramids, and cone.

- Spheres: are solid shapes e.g. a ball.
- Prism is a solid shape with a uniform cross section
- Pyramid is a solid shape without uniform cross section.

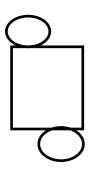
### **Difference between Plane Shapes and Solid Shapes**

Plane Shapes	Solid Shape	
- Two lines meet a point call vertex	More than two line meet at a point	
- Two dimentional shape	Three dimentional shape.	

**Note:** For each solid shape, ask pupils to determine the number of faces, the number of edges and the number of vertices and record their result in a table as shown below:

Name of Solid	Number of faces	Number of vertices	Number of Edges
Shape			
Cube	6	8	12
Cuboid			
Triangular			
Square pyramid			
Triangular pyramid			

- Ask pupils to find out if the relationship E = F + V 2 holds for each solid shape: where
- E, F and V are the number of edges, number of faces and number of vertices respectively



Net of Cylinder



Net of Cone

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