E.P COLLEGE OF EDUCATION

UNIVERSITY OF GHANA AFFILIATION

FOUR YEAR BACHELOR OF EDUCATION

END OF SEMESTER EXAMINATION

COURSE TITLE: LEARNING, TEACHING AND APPLYING FURTHER ALGEBRA

COURSE CODE: MAE 214

1. Expand $(2x + 3)^3$ using the binomial theorem.

   A. $2x^3 + 36x^2 + 54x + 27$

   B. $2x^3 + 18x^2 + 54x + 27$

   C. $8x^3 + 27x^2 + 54x + 9$

   D. $8x^3 + 36x^2 + 54x + 27$

2. Find the reminder when $x^3 - x^2 + 6$ is divided by $(x + 2)$

   A. -12

   B. -6

   C. 6

   D. 12

3. Evaluate the determinant, $\begin{vmatrix} 2 & -4 \\ \frac{1}{2} & 3 \end{vmatrix}$

   A. -8

   B. 4

   C. 8

   D. 10

4. If the general term of a sequence is $\frac{n-1}{n+1}$, find the 5th term

   A. $\frac{1}{3}$

   B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{5}{6}$

5. Find the common ratio for the exponential sequence with $a_2 = 24 \; and \; a_5 = 648$

A. $\frac{3}{2}$

B. 2

C. 3

D. 4

6. Find the truth set of $\frac{2}{x} + 3 = \frac{5}{2x}$

A. -6

B. 0

C. $\frac{1}{6}$

D. 6

7. One factor of a polynomial $P(x)$ is $x^2 - 5x - 12$, and $P(-3) = 0. Find \; P(x)$

A. $x^3 - 8x^2 - 27x - 36$

B. $x^3 + 8x^2 - 27x - 36$

C. $x^3 - 2x^2 - 27x - 36$

D. $x^3 - 2x^2 + 27x - 36$

8. A binary operation is defined by a*b = $a^2 - b^2 + ab, where \; a \; and \; b$ are real numbers. Evaluate $\sqrt{2} * \sqrt{3}$

A. $1 - \sqrt{6}$

B. $\sqrt{6} - 1$

C. $\sqrt{6}$

D. $\sqrt{6} + 1$

9. The area of a triangle with base length $(x + 3y)$ is $(x^2 + 2xy - 3y^2)$. Determine the height of the triangle in terms of $x \; and \; y$.

A. $2x - y$

B. $x - 2y$

C. $y - 2x$

D. $2(x - y)$

10. The houses along a street in a certain city are assigned odd numbers, starting from 7. If the last house is numbered 141. Find the number of houses on that street?

A. 63

B. 66

C. 68

D. 136

11. Find the value of k such that the determinant of the matrix $\begin{pmatrix} 4 & 6 \\ -2 & k \end{pmatrix}$ is 24.

A. -9

B. -3

C. 3

D. 9

12. Find the values of x that satisfy the inequality $\frac{4-3x}{3} \geq \frac{1}{2}x - 3$.

A. $x \leq \frac{11}{9}$

B. $x \leq \frac{26}{9}$

C. $x \geq \frac{11}{9}$

D. $x \geq \frac{26}{9}$

13. Find the 20th term of the sequence defined by $a_n = n^2 - n$

A. 39

B. 199

C. 380

D. 399

14. Which of the following expressions is the reduced form of $\frac{x^2-x-6}{3x+6}$?

A. $\frac{1}{3}(x - 3)$

B. $\frac{1}{2}(x - 3)$

C. $\frac{1}{3}(x+3)$

D. $\frac{1}{2}(x+3)$

15. Find the common ratio for the exponential sequence with $a_2 = 10\ and\ a_4 = 40$

A. 2

B. 3

C. 6

D. 9

16. Find the value of y such that the determinant of the matrix $\begin{pmatrix} y & 7 \\ 3 & 2 \end{pmatrix}$ is 3

A. -9

B. 9

C. 12

D. 24

17. A binary operation Δ, is defined on the set R, of real numbers by $a \, \Delta \, b = \frac{a}{b} - \frac{b}{a}$ where a, b ∊ R and a, b ≠ 0, Evaluate 2 Δ $\sqrt{2}$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{2}{\sqrt{2}}$

C. $\frac{\sqrt{2}}{4}$

D. $\frac{\sqrt{2}}{2}$

18. Which of the following logarithmic statement is true?

A. $\frac{log5}{log3} = \log\left(\frac{5}{3}\right)$

B. $(log_2 y)^3 = 3log_2 y$

C. $log_a(x+y) = log_a x + log_a y$

D. $log_a(xy) = log_a x + log_a y$

19. Given the matrices A = $\begin{pmatrix} -2 & 5 \\ 4 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 4 & -3 \\ -6 & 5 \end{pmatrix}$. Find AB

A. $\begin{pmatrix} -38 & 31 \\ -2 & 3 \end{pmatrix}$

B. $\begin{pmatrix} -20 & -11 \\ 32 & -15 \end{pmatrix}$

C. $\begin{pmatrix} -20 & 11 \\ 32 & -15 \end{pmatrix}$

D. $\begin{pmatrix} -20 & -11 \\ 32 & -15 \end{pmatrix}$

20. Simplify $\frac{2}{1-y} + \frac{2}{1+y}$

A. $\frac{2}{1+y^2}$

B. $\frac{2}{1-y^2}$

C. $\frac{4}{1+y^2}$

D. $\frac{4}{1-y^2}$

21. Solve for x in the equation $3^{x-2} = 7$, correct to six decimal places.

A. 0.771244

B. 1.77123

C. 2.771243

D. 3.771244

22. Solve for $x$in the logarithmic equation, $log_2(25 - x) = 4$.

A. 8

B. 9

C. 17

D. 19

23. A binary operation is defined by $x * y = x^2 - y^2 + xy, where\ x\ and\ y$ are real numbers. Evaluate $\sqrt{5} * 2$.

A. $\sqrt{5} - 4$

B. $5 - 2\sqrt{5}$

C. $\sqrt{5} + 1$

D. $1 + 2\sqrt{5}$

24. Find the seventh term of the exponential sequence $3, -\frac{3}{2}, \frac{3}{4}, \ldots.$

A. $\frac{3}{32}$

B. $\frac{7}{32}$

C. $\frac{3}{64}$

D. $\frac{7}{64}$

25. If $log_3 5 = 1.465$, find the value of $log_3 25 + log_3 15$.

A. 3.930

B. 4.395

C. 4.930

D. 5.395

Section B

1. A binary operations $*$ is defined on the set R of real numbers by $a * b = \frac{a}{b} - \frac{b}{a}$ where $a, b \in R$ and $a, b \neq 0$.

(i) Calculate $(3\sqrt{2} - 2\sqrt{3}) * (\sqrt{3} - \sqrt{2})$. **(7 marks)**

(ii) Deduce the value of $(\sqrt{3} - \sqrt{2}) * (3\sqrt{2} - 2\sqrt{3})$. **(7 marks)**

(iii) (i) If $a * b = 1$, show that $b = \frac{1}{2}(-a \pm a\sqrt{3})$. **(6 marks)**

(iv) Hence find the values of b for which $(4 - 2\sqrt{3}) * b = 1$. Give your answer in the form $p + q\sqrt{r}$ where p, q and r are rational numbers. **(5 marks)**

2. The polynomial $f(x) = x^3 - x^2 - 6kx + 4k^2$, where k is a contant has $(x - 3)$ as a factor.

(i) Find the possible value of k **(7 marks)**

(ii) For the integral value of k obtained in (a), find the remainder when $f(x)$ is divided by $(x + 2)$. **(5 marks)**

(b) A factory produced 360 tons of plastic goods in 2001. Since then, it increased its production by 10% every year.

(i) Write down the number of tons produced for the next three years. **(3 marks)**

(ii) Determine the number of tons produced in 2008. **(5 marks)**

(iii) Calculate the total production of the factory from 2001 to 2010 **(5 marks)**

3.(a) Solve the system of linear equations using matrix approach and the Cramer's rule.

$$\begin{cases} 5x - 3y = 28 \\ -2x + 4y = 14 \end{cases}$$

(b) Find the sum of the first n terms of the sequence $k, k+s, k+2s, k+3s, \dots, k+(n-1)s$.

(c) How many consecutive counting numbers beginning from one must be added in order to obtain 1326?

4. (a) Given that $\boldsymbol{Z} = \begin{pmatrix} 3 & -2 & 0 \\ 1 & -4 & -5 \\ 2 & 1 & -3 \end{pmatrix}$ and $\boldsymbol{F} = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \\ 3 & -2 & 7 \end{pmatrix}$

Evaluate

(i) $\boldsymbol{ZF}$ **(5 marks)**

(ii) $\boldsymbol{FZ}$ **(5 marks)**

(iii) What conclusion can you draw from the result in (i) and (ii) above? **(2 marks)**

(b) (i)Use the expansion of $(\mathbf{1 - 5x})^{\mathbf{5}}$ to estimate the value of 0.995, correct to 3 significant figures. **(7 marks)**

(ii) Using Pascal's coefficient, expand and simplify $[(2x+1) - y]^3$ **(6 marks)**

5. (a) Solve the equation $2^{2x} - 9(2^x) + 20 = 0$ **(8 marks)**

(b) The sum of the first n terms of an arithmetic progression is 21, and the seventeenth term is three times the sum of the third and fourth. Find the first term and the common difference. **(8 marks)**

(c) Show that the sum of the first n terms of an arithmetic progression with the first term a and common difference of d is given by $S_n = \frac{n}{2}\{2a + (n-1)d\}$.

(9 marks)

SECTION A

1. D
2. B
3. C
4. C
5. C
6. C
7. C
8. B
9. D
10. C
11. C
12. D
13. C
14. A
15. A
16. C
17. D
18. D
19. A
20. D
21. D
22. B
23. D
24. C
25. D

Solution: (1) $a * b = \frac{a}{b} - \frac{b}{a}$

(i)$\left(3\sqrt{2} - 2\sqrt{3}\right) * \left(\sqrt{3} - \sqrt{2}\right) = \frac{\left(3\sqrt{2}-2\sqrt{3}\right)}{\left(\sqrt{3}-\sqrt{2}\right)} - \frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(3\sqrt{2}-2\sqrt{3}\right)}$, rationalizing the denominator

$$\frac{3\sqrt{2}-2\sqrt{3}}{\sqrt{3}-\sqrt{2}} = \frac{\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$$

$$= \frac{3\sqrt{6}+3\sqrt{4}-2\sqrt{9}-2\sqrt{6}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{6}+6-6}{3-2} = \frac{\sqrt{6}}{1} = \sqrt{6}$$

$$\frac{\sqrt{3}-\sqrt{2}}{3\sqrt{2}-2\sqrt{3}} = \frac{\left(\sqrt{3}-\sqrt{2}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}$$

$$= \frac{3\sqrt{6}+6-6-2\sqrt{6}}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{6}}{18-12} = \frac{\sqrt{6}}{6}$$

$$\therefore \left(3\sqrt{2} - 2\sqrt{3}\right) * \left(\sqrt{3} - \sqrt{2}\right) = \sqrt{6} - \frac{\sqrt{6}}{6} = \frac{5}{6}\sqrt{6}$$

(ii) To deduce the value of $\left(3\sqrt{2} - 2\sqrt{3}\right) * \left(\sqrt{3} - \sqrt{2}\right)$ we see from (i) that the positions of $\left(\sqrt{3} - \sqrt{2}\right)$ and $\left(3\sqrt{2} - 2\sqrt{3}\right)$ have been interchanged i.e.

From $a * b = \frac{a}{b} - \frac{b}{a}$.

$$\Rightarrow b * a = \frac{b}{a} - \frac{a}{b} = -\left(\frac{a}{b} - \frac{b}{a}\right) = -(a * b)$$

Hence, $\left(\sqrt{3} - \sqrt{2}\right) * \left(3\sqrt{2} - 2\sqrt{3}\right) = -\left[\left(3\sqrt{2} - 2\sqrt{3}\right) * \left(\sqrt{3} - \sqrt{2}\right)\right]$

$$= -\frac{5}{6}\sqrt{6}$$

Since $\left(\sqrt{3} - \sqrt{2}\right) * \left(3\sqrt{2} - 2\sqrt{3}\right) = \frac{5}{6}\sqrt{6}$, from a) (i) above.

(iii) Given that $a * b = 1$

$\Rightarrow \frac{a}{b} - \frac{b}{a} = 1$, solving for b, we have

$\Rightarrow \frac{a^2 - b^2}{ab} = 1$

$\Rightarrow a^2 - b^2 = ab$

$\Rightarrow b^2 + ab - a^2 - 0 \ldots\ldots\ldots\ldots..(1)$, a quadratic equation in b,

$b = \frac{-a \pm \sqrt{a^2 - 4(1)(-a)^2}}{2(1)}$

$= \frac{-a \pm \sqrt{a^2 + 4a^2}}{2}$

$= \frac{-a \pm \sqrt{5a^2}}{2} = \frac{1}{2}\left(-a \pm a\sqrt{5}\right)$

(iv) Comparing $\left(4 - 2\sqrt{5}\right) * b = 1$ with $a * b = 1, \Rightarrow a = 4 - 2\sqrt{5}$.

From (i) above, $b = \frac{1}{2}\left(-a \pm a\sqrt{5}\right)$

$\Rightarrow b = \frac{1}{2}\left\{-\left(4 - 2\sqrt{5}\right) + \left(4 - 2\sqrt{5}\right)\sqrt{5}\right\}$

The two values of b are given by each sign.

i.e. $b_1 = \frac{1}{2}\left\{\left(-4 + 2\sqrt{5}\right) + \left(4\sqrt{5} - 10\right)\right\}$

$= \frac{1}{2}\left(-14 - 6\sqrt{5}\right) = -7 + 3\sqrt{5}$ and

$b_2 = \frac{1}{2}\left\{\left(-4 + 2\sqrt{5}\right) - \left(4\sqrt{5} - 10\right)\right\}$

$= \frac{1}{2}\left\{6 - 2\sqrt{5}\right\} = 3 - \sqrt{5}$

$\therefore b = -7 + 3\sqrt{5}$ or $3 - \sqrt{5}$

2. (i) Since $(x - 3)$ is a factor of $x^3 - x^2 - 6kx + 4k^2$, then by the factor theorem $f(3) = 0$.

$f(3) = 3^3 - 3^2 - 6k(3) + 4k^2 = 0$

$= 27 - 9 - 18k + 4k^2 = 0$

$= 18 - 18k + 4k^2 = 0$, Divided through by 2

$\Rightarrow 2k^2 - 9k + 9 = 0$

$\Rightarrow 2k^2 - 6k - 3k + 9 = 0$

$\Rightarrow 2k(\ k-3)-3\ (k-3)=0$

$\Rightarrow 2k\ (k-3)(\ k-3)=0$

$\Rightarrow 2k - 3 = 0,\ k - 3 = 0$

$\Rightarrow k = {}^{3}/_{2}\ \ or\ \ k=3$

(ii) From part (a) , the integral value of k is 3. Hence , substituting $k=3$ into $f(x)$, we have $f(x)=x^3-\ x^2-6(3)x\ +4(3)^2$

$= \ x^3-\ x^2-18x+36$

By the remainder theorem, when $f(x)=\ x^3-\ x^2-18x+36$ is divided by $(\ x+2)$, we have $f(-2)=$ $(-2)^3-\ (-2)^2-18(-2)+36$

$=-8-4+36+36$

$= 60$

∴ The remainder is 60

(c) 1st year 2nd year 3rd year, ---, 8th year

360 396 435.6, ---, Total production , i.e. 10% of

$360\ =\ 0.1\times 360=36+360=396$ and 10% of

$396\ =\ 0.1\times 396=39.6+396=435.6.$

The sequence is a G.P with the first term, $a=360$, and common ratio;

$r=\frac{396}{360}=\frac{435.6}{396}=\frac{11}{10}=1.1$

(ii) In 2008, $n\ =\ 8,$

$U_n=ar^{n-1}$

$=360\times 1.1^{8-1}$

$=360\times 1.9487171\ =\ 701.5$

(iii) From 200 to 2010 there are 10 years $\Rightarrow n=10$

Sum of a G.P is:

$Sn=\frac{a(r^n-1)}{r-1}$, since $r>1$.

$S_8=\frac{360[(1.1)^{10}-1]}{1.1-1}=5737.47$

$\therefore$ The total production from 2003 to 2010 is 5737.47 tons

3. (a) $\begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 28 \\ 14 \end{pmatrix}$

Let $A = \begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}$

$\det A = 5(4) — 2\,(-3) = 20 - 6 = 14$

$|A_x| = \begin{vmatrix} 28 & -3 \\ 14 & 4 \end{vmatrix} = 28(4) - 14(-3) = 154$

$|A_y| = \begin{vmatrix} 5 & 28 \\ -2 & 14 \end{vmatrix} = 5(14) - 28(-2) = 126$

The solution is therefore given by $x = \frac{|A_x|}{|A|} = \frac{154}{4} = 11$ and $y = \frac{|A_y|}{|A|} = \frac{126}{14} = 9$

Therefore $x = 11$ and $y = 9$.

(b) $S_n = k + (k+s) + (k+2s) + (k+3s) + \cdots + k + (n-1)s$ ------------------ (1)

$\Rightarrow S_n = k + (n-1)s + k + (n-1)s + \cdots + k$ --- (2)

$(1) + (2)$: $2S_n = 2k + (n-1)s + 2k + (n-1)s + \cdots + 2k + (n-1)s$

$\Rightarrow 2S_n = n[2k + (n-1)s]$

$\therefore S_n = \frac{n}{2}[2k + (n-1)s]$

(c) Counting numbers: $1, 2, 3, 4, \ldots,$ This is an A.P with $a = 1$, $d = 1$ and $S_n = 1326$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 1326 = \frac{n}{2}[2(1) + (n-1)1]$

$\Rightarrow 26552 = n(2 + n - 1)$

$\Rightarrow 2652 = n(n+1)$

$\Rightarrow 2652 = n^2 + n$

$\Rightarrow n^2 + n - 2652 = 0$, where $a = 1, b = 1$ and $c = -2652$

Using the formula $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-2652)}}{2(1)}$

$= \frac{-1 \pm \sqrt{1 + 10608}}{2}$

$$= \frac{-1 \pm \sqrt{10609}}{2}$$

$= \frac{-1 \pm 103}{2}$, and taking the positive root,

$$n = \frac{-1+103}{2} = \frac{102}{2} = 51$$

Hence, there are 51 terms.

4. **Solution: (a)**

(i) $$\boldsymbol{ZF} = \begin{pmatrix} 3 & -2 & 0 \\ 1 & -4 & -5 \\ 2 & 1 & -3 \end{pmatrix}\begin{pmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \\ 3 & -2 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 3(1) - 2(2) + 0(3) & 3(4) - 2(3) + 0(-2) & 3(5) - 2(6) + 0(7) \\ 1(1) - 4(2) - 5(3) & 1(4) - 4(3) - 5(-2) & 1(5) - 4(6) - 5(7) \\ 2(1) + 1(2) - 3(3) & 2(4) + 1(3) - 3(-2) & 2(5) + 1(6) - 3(7) \end{pmatrix}$$

$$= \begin{pmatrix} 3 - 4 + 0 & 12 - 6 + 0 & 15 - 12 + 0 \\ 1 - 8 - 15 & 4 - 12 + 10 & 5 - 24 - 35 \\ 2 + 2 - 9 & 8 + 3 + 6 & 10 + 6 - 2 \end{pmatrix}$$

$$\therefore \boldsymbol{ZF} = \begin{pmatrix} -1 & 6 & 3 \\ -22 & 2 & -54 \\ 3 & 17 & -5 \end{pmatrix}$$

(ii) $$\boldsymbol{FZ} = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \\ 3 & -2 & 7 \end{pmatrix}\begin{pmatrix} 3 & -2 & 0 \\ 1 & -4 & -5 \\ 2 & 1 & -3 \end{pmatrix} = \begin{pmatrix} 17 & -13 & -35 \\ 21 & -10 & -33 \\ 21 & 9 & -11 \end{pmatrix}$$

(iii) matrix multiplication is not **commutative** i.e. $\boldsymbol{ZF} \neq \boldsymbol{FZ}$.

(b) (i) $(1 - 5x)^5 = 1 + 5(-5x) + \frac{5(4)}{2}(-5x)^2 + \frac{5(4)(3)}{6}(-5x)^3 + \cdots$

$$= 1 - 25x + 250x^2 - 1250x^3 + \cdots$$

Comparing, $(0.995)^5 = (1 - 0.005)^5$. Comparing; $5x = 0.005, x = 0.001$

$$\therefore (0.995)^5 = 1 - 25(0.001) + 250(0.001)^2 - 1250(0.001)^3 + \cdots$$

$$= 1 - 0.025 + 0.00025 - 0.00000125$$

$$= 0.97524875$$

$\therefore (0.995)^5 \cong 0.975$ (3.s.f)

(ii) $[(2x + 1) - y]^3$ using the Pascal coefficients: $1, 3, 3, 1$ and let $2x + 1 = a$,

$[(2x + a) - y]^3 \equiv (a - y)^3$

$$= a^3 + a^2(-y) + a(-y^2) + (-y^3)$$

$$= a^3 - a^2y + ay^2 - y^3$$

$$= a^3 - 3a^2y + 3ay^2 - y^3$$

But $a = 2x + 1[(2x + 1)]^3$

$$= (2x + 1)^3 - 3(2x + 1)^2y + 3(2x + 1)y^2 - y^3$$

$$= 8x^3 + 12x^2 + 6x + 1 - 3(4x^2 + 4x + 1)y + (6x + 3)y^2 - y^3$$

$$= 8x^3 + 12x^2 + 6x + 1 - 3(4x^2y + 4xy + y) + 6xy^2 + 3y^2 - y^3$$

$$= 8x^3 + 12x^2 + 6x + 1 - 12x^2y - 12xy - 3y + 6xy^2 + 3y^2 - y^3$$

$$\therefore [(2x + 1) - y]^3 = 8x^3 + 12x^2 + 6x - 12x^2y - 12xy + 6xy^2 + 3y^2 - 3y - y^3 + 1$$

5. (a) $2^{2x} - 9(2^x) + 20 = 0$

$$Let\ y = 2^x$$
$$\Rightarrow y^2 - 9y + 20 = 0$$

$\Rightarrow y^2 - 4y - 5y + 20 = 0$

$$\Rightarrow y(y - 4) - 5(y - 4) = 0$$
$$\Rightarrow (y - 4)(y - 5) = 0$$
$$\therefore y = 4\ or\ y = 5$$

But $y = 2^x$

$$\therefore 2^x = 4\ or\ 5$$

Now $2^x = 4 = 2^2, \Rightarrow x = 2$

Again, $2^x = 5$

Take log of both sides to base ten.

$$\Rightarrow log_{10}2^x = log_{10}5$$

$$\Rightarrow xlog_{10}2 = log_{10}5$$

$$\therefore x = \frac{log_{10}5}{log_{10}2} = \frac{0.6989}{0.3010} = 2.32$$

Hence, $x = 2\ or\ 2.32$

(b) $S_n = a + (a + d) + (a + 2d) + \cdots + a + (n - 2)d + a + (n - 1)d \ldots \ldots (1)$, Reversing (1):

$$S_n = a + (n - 1)d + a + (n - 2)d + \cdots + (a + 2d) + (a + d) + a \ldots . (2)$$

$$(1) + (2): 2S_n = 2a + (n - 1)d + \cdots + 2a + (n - 1)d + 2a + (n - 1)d$$

$$2S_n = n(2a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

(c) $U_7 = a + 6d,\ U_3 = a + 2d$ and $U_4 = a + 3d$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$21 = \frac{6}{2}[2a + (6-1)d]$$

$$21 = 6a + 15d \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Again, $a + 6d = 3[a + 2d + a + 3d]$

$$a + 6d = 3(2a + 5d)$$

$$-5a = 9d$$

$$a = \frac{-9}{5}d \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Put (2) into (1): $21 = 6\left(\frac{-9}{5}d\right) + 15d$

$$105 = -54d + 75d$$

$$d = \frac{105}{21} = 5$$

Put $d = 5$ into (2): $a = \frac{-9}{5}(5) = -9$